Chapter 5 The Trigonometric Functions

5-1 Angles and Degree Measure

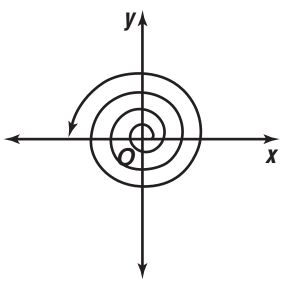
Pages 280–281 Check for Understanding

1. If an angle has a positive measure, the rotation is in a counterclockwise direction. If an angle has a negative measure, the rotation is in a clockwise direction.

2. Add 29, $\frac{45}{60}$, and $\frac{26}{3600}$.

3. $270° + 360k°$ where k is an integer

4.



1260°

5. $34.95° = 34° + (0.95 \cdot 60)'$
$= 34° + 57'$
$34° 57'$

6. $-72.775° = -(72° + (0.775 \cdot 60)')$
$= -(72° + 46.5')$
$= -(72° + 46' + (0.5 \cdot 60)'')$
$= -(72° + 46' + 30'')$
$-72° 46' 30''$

7. $-128° 30' 45' = -\left(128° + 30'\left(\frac{1°}{60'}\right) + 45''\left(\frac{1°}{3600''}\right)\right)$
$= -128.513°$

8. $29° 6' 6'' = 29° + 6'\left(\frac{1°}{60'}\right) + 6''\left(\frac{1°}{3600''}\right)$
$= 29.102°$

9. $2 \times (-360°) = -720°$

10. $4.5 \times 360° = 1620°$

11. $22° + 360k°$; Sample answers:
$22° + 360k° = 22° + 360(1)°$ or $382°$
$22° + 360k° = 22° + 360(-1)°$ or $-338°$

12. $-170° + 360k°$; Sample answers:
$-170° + 360k° = -170° + 360(1)°$ or $190°$
$-170° + 360k° = -170° + 360(-1)°$ or $-530°$

13. $\frac{453}{360} \approx 1.26$
$\propto + 360(1)° = 453°$
$\propto + 360 = 453°$
$\propto = 93°$; II

14. $-\frac{798}{360} \approx -2.22$
$-2.22 + 2 = -0.22$
$-0.22 \times 360° = -78°$
$360° - 78° = 282°$; IV

15. $\propto - 180° = 227° - 180°$
$= 47°$

16. $360° - 210° = 150°$
$180° - \propto = 180° - 150°$
$= 30°$

17. $\frac{1}{24}(360°) = 15°$
$\frac{1}{60}\left(\frac{1}{24}\left(360°\right)\right) = 0.25°$, or $0.25(60') = 15'$
$\frac{1}{60}\left(\frac{1}{60}\left(\frac{1}{24}\left(360°\right)\right)\right) \approx 0.0042°$,
or $0.0042(60)(60) = 15''$

Pages 281–283 Exercises

18. $-16.75° = -(16° + (0.75 \cdot 60)')$
$= -(16° + 45')$
$-16° 45'$

19. $168.35° = 168° + (0.35 \cdot 60)'$
$= 168° + 21'$
$168° 21'$

20. $-183.47° = -(183° + (0.47 \cdot 60)')$
$= -(183° + 28.2')$
$= -(183° + 28' + (0.2 \cdot 60''))$
$= -(183° + 28' + 12'')$
$-183° 28' 12''$

21. $286.88° = 286° + (0.88 \cdot 60)'$
$= 286° + 52.8'$
$= 286° + 52' + (0.8 \cdot 60)''$
$= 286° + 52' + 48''$
$286° 52' 48''$

22. $27.465° = 27° + (0.465 \cdot 60)'$
$= 27° + 27.9'$
$= 27° + 27' + (0.9 \cdot 60)''$
$= 27° + 27' + 54''$
$27° 27' 54''$

23. $246.876° = 246° + (0.876 \cdot 60)'$
$= 246° + 52.56'$
$= 246° + 52' + (0.56 \cdot 60)''$
$= 246° + 52' - 33.6''$
$246° 52' 33.6''$

24. $23° 14' 30'' = 23° + 14'\left(\frac{1°}{60'}\right) + 30''\left(\frac{1°}{3600''}\right)$
$= 23.242°$

25. $-14° 5' 20'' = -\left(14° + 5'\left(\frac{1°}{60'}\right) + 20''\left(\frac{1°}{3600''}\right)\right)$
$= -14.089°$

26. $233° 25' 15'' = 233° + 25'\left(\frac{1°}{60'}\right) + 15''\left(\frac{1°}{3600''}\right)$
$= 233.421°$

27. $173° 24' 35'' = 173° + 24'\left(\frac{1°}{60'}\right) + 35''\left(\frac{1°}{3600''}\right)$
$= 173.410°$

28. $-405° 16' 18'' = -\left(405° + 16'\left(\frac{1°}{60'}\right) + 18''\left(\frac{1°}{3600''}\right)\right)$
$= -405.272°$

29. $1002° 30' 30'' = 1002° + 30'\left(\frac{1°}{60'}\right) + 30''\left(\frac{1°}{3600''}\right)$
$= 1002.508°$

30. $3 \times -360° = -1080°$ **31.** $2 \times 360° = 720°$

32. $1.5 \times 360° = 540°$ **33.** $7.5 \times (-360°) = -2700°$

34. $2.25 \times 360° = 810°$ **35.** $5.75 \times (-360°) = -2070°$

36. $4 \times 360° = 1440°$

37. $30° + 360k°$; Sample answers:
$30° + 360k° = 30° + 360(1)°$ or $390°$
$30° + 360k° = 30° + 360(-1)°$ or $-330°$

38. $-45° + 360k°$; Sample answers:
$-45° + 360k° = -45° + 360(1)°$ or $315°$
$-45° + 360k° = -45° + 360(-1)°$ or $-405°$

39. $113° + 360k°$; Sample answers:
$113° + 360k° = 113° + 360(1)°$ or $473°$
$113° + 360k° = 113° + 360(-1)°$ or $-247°$

40. $217° + 360k$; Sample answers:
$217° + 360k° = 217° + 360(1)°$ or $577°$
$217° + 360k° = 217° + 360(-1)°$ or $-143°$

41. $-199° + 360k°$; Sample answers:
$-199° + 360k° = -199° + 360(1)°$ or $161°$
$-199° + 360k° = -199° + 360(-1)°$ or $-559°$

42. $-305° + 360k°$; Sample answers:
$-305° + 360k° = -305° + 360(1)°$ or $55°$
$-305° + 360k° = -305° + 360(-1)°$ or $-665°$

43. $310° + 360k° = 310° + 360(0)°$ or $310°$

44. $60° + 360k° = 60° + 360(2)°$ or $780°$
$60° + 360k° = 60° + 360(-3)$ or $-1020°$

45. $\frac{400°}{360°} \approx 1.11$
$\propto + 360(1)° = 400°$
$\propto + 360° = 400°$
$\propto = 40°$; I

46. $\frac{-280°}{360°} \approx -0.78$
$\propto + 360(-1)° = -280°$
$\propto - 360° = -280°$
$\propto = 80°$; I

47. $\frac{940°}{360°} \approx 2.61$
$\propto + 360(2)° = 940°$
$\propto + 720° = 940°$
$\propto = 220°$; III

48. $\frac{1059°}{360°} \approx 2.94$
$\propto + 360(2)° = 1059°$
$\propto + 720° = 1059°$
$\propto = 339°$; IV

49. $\frac{-624°}{360°} \approx -1.73$
$-1.73 + 1 = -0.73$
$-0.73 \times 360° = -264°$
$360° - 264° = 96°$; II

50. $\frac{-989°}{360°} \approx -2.75$
$-2.75 + 2 = -0.75$
$-0.75 \times 360° = -269°$
$360° - 269° = 91°$; II

51. $\frac{1275°}{360°} \approx 3.54$
$\propto + 360(3)° = 1275°$
$\propto + 1080° = 1275°$
$\propto = 195°$; III

52. $360° - \propto = 360° - 327°$ or $33°$

53. $180° - \propto = 180° - 148°$ or $32°$

54. $563° - 360° = 203°$
$\propto - 180° = 203° - 180°$ or $23°$

55. $-420° + 360° = -60°$
$360° - 60° = 300°$
$360° - 300° = 60°$

56. $360° - 197° = 163°$
$180° - 163° = 17°$

57. $\frac{1045°}{360°} \approx 2.90$
$\propto + 360(2)° = 1045°$
$\propto + 720° = 1045°$
$\propto = 325°$
$360° - \propto = 360° - 325°$ or $35°$

58. $20°$
$180° - \propto = 180° - 20°$ or $160°$
$180° + \propto = 180° + 20°$ or $200°$
$360° - \propto = 360° - 20°$ or $340°$

59. $\frac{90 \text{ revolutions}}{\text{second}} \cdot \frac{360°}{\text{revolution}} = 32{,}400°/\text{second}$
$\frac{90 \text{ revolutions}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{360°}{\text{revolution}} =$
$1{,}944{,}000°/\text{minute}$

60. $90k$, where k is an integer

61. $\frac{95 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} = 34{,}200°/\text{minute}$
$30 \text{ seconds} = \frac{1}{2} \text{ minute}$
$34{,}200° \cdot \frac{1}{2} = 17{,}100°$

62. $\frac{30{,}000 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} =$
$10{,}800{,}000$ or 1.08×10^7
$\frac{100{,}000 \text{ revolutions}}{\text{minute}} \cdot \frac{360°}{\text{revolution}} =$
$36{,}000{,}000$ or 3.6×10^7
1.08×10^7 to 3.6×10^7 degrees

63. $\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} = 22{,}320° \text{ second}$
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} =$
$1{,}339{,}200°/\text{minute}$
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} =$
$80{,}352{,}000°/\text{hour}$
$\frac{62 \text{ rotations}}{\text{second}} \cdot \frac{360°}{\text{rotation}} \cdot \frac{60 \text{ seconds}}{\text{minute}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot$
$24 \frac{\text{hours}}{\text{day}} = 1{,}928{,}448{,}000°/\text{day}$

64. $25° + 120k°$, where k is an integer

65a. $44.4499° = 44° + (0.4499 \cdot 60)'$
$= 44° + 26.994'$
$= 44° + 26' + (0.994 \cdot 60)''$
$= 44° + 26' + 59.64''$
$44° \ 26' \ 59.64''$
$68.2616° = 68° + (0.2616 \cdot 60)'$
$= 68° + 15.696'$
$= 68° + 15' + (0.696 \cdot 60)''$
$= 68° + 15' + 41.76''$
$68° \ 15' \ 41.76''$

65b. $24° \ 33' \ 32'' = 24° + 33'\left(\frac{1°}{60'}\right) + 32''\left(\frac{1°}{3600''}\right)$
$\approx 24.559°$
$81° \ 45' \ 34.4'' = 81° + 45'\left(\frac{1°}{60'}\right) + 34.4''\left(\frac{1°}{3600''}\right)$
$\approx 81.760°$

66a. Sydney:
$\frac{1 \text{ rotation}}{70 \text{ minutes}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot \frac{24 \text{ hours}}{\text{day}} \approx \frac{20.6 \text{ rotations}}{\text{day}}$
San Antonio: $\frac{1 \text{ revolution}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{\text{day}} = \frac{24 \text{ revolutions}}{\text{day}}$
$24 - 20.6 = 3.4$ revolutions
about 3.4 revolutions

66b. Sydney:
$\frac{20.6 \text{ rotations}}{\text{day}} \cdot \frac{7 \text{ days}}{\text{week}} \cdot \frac{360°}{\text{rotation}} = 51{,}840°$
San Antonio: $\frac{24 \text{ revolutions}}{\text{day}} \cdot \frac{7 \text{ days}}{\text{week}} \cdot \frac{360°}{\text{revolution}} =$
$60{,}480°$
$60{,}480° - 51{,}840° = 8640°$

67a. Use graphing calculator to find cubic regression.
Sample answer: $f(x) = -0.00055x^3 + 0.0797x^2$
$- 3.7242x + 76.2147$

67b. $2010 - 1950 = 60$
$f(x) = -0.00055x^3 - 0.0797x^2 - 3.7242x$
$+ 76.2147$
$f(60) = -0.00055(60)^3 + 0.0797(60)^2$
$- 3.7242(60) + 76.2147$
$= 20.8827$
Sample answer: about 20.9%

68. $\sqrt[3]{6n+5} - 15 = -10$

$$\begin{aligned} \sqrt[3]{6n+5} &= 5 \\ 6n + 5 &= 125 \\ 6n &= 120 \\ n &= 20 \end{aligned}$$

Check: $\sqrt[3]{6n+5} - 15 = -10$

$$\begin{aligned} \sqrt[3]{6(20)+5} - 15 &\stackrel{?}{=} -10 \\ \sqrt[3]{125} - 15 &\stackrel{?}{=} -10 \\ 5 - 15 &= -10 \\ -10 &= -10 \quad \checkmark \end{aligned}$$

69.
$$\begin{aligned} \frac{x+3}{x+2} &= 2 - \frac{3}{x^2+5x+6} \\ \frac{x+3}{x+2} &= 2 - \frac{3}{(x+2)(x+3)} \\ (x+2)(x+3)\left(\frac{x+3}{x+2}\right) &= (x+2)(x+3)(2) - \\ &\quad (x+2)(x+3)\left(\frac{3}{(x+2)(x+3)}\right) \\ (x+3)(x+3) &= (x+2)(x+3)(2) - 3 \\ x^2 + 6x + 9 &= 2x^2 + 10x + 12 - 3 \\ 0 &= x^2 + 4x \\ 0 &= x(x+4) \end{aligned}$$

$x = 0$ or $x + 4 = 0$

$x = -4$

70.

2	1	1	0	8	1
			2	4	24
		1	2	12	25

25

71.
$$\begin{aligned} (x-(-5))(x-(-6))(x-10) &= 0 \\ (x+5)(x+6)(x-10) &= 0 \\ (x^2+11x+30)(x-10) &= 0 \\ x^3 + x^2 - 80x - 300 &= 0 \end{aligned}$$

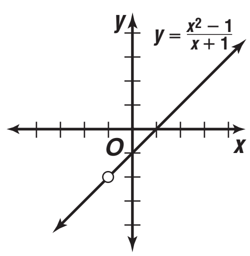
72.
$$\begin{aligned} r_1t_1 &= r_2t_2 \\ 18(-3) &= r_2(-11) \\ \frac{18(-3)}{-11} &= r_2 \\ 4.91 &\approx r_2 \end{aligned}$$

about 4.91

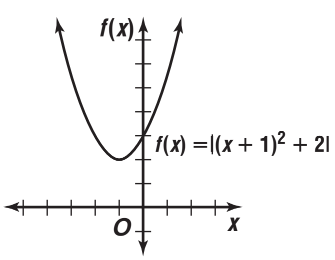
73. $x + 1 = 0$

$x \neq -1$

Point discontinuity



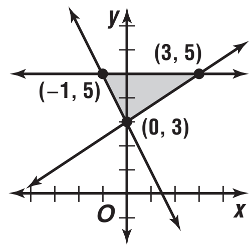
74.



decreasing for $x < -1$, increasing for $x > -1$

75. expanded vertically by a factor of 3, translated down 2 units

76.



77.
$$\begin{aligned} [f \cdot g](x) &= f(g(x)) \\ &= f(x - 0.3x) \\ &= (x - 0.3x) - 0.2(x - 0.3x) \\ &= x - 0.3x - 0.2x + 0.06x \\ &= 0.56x \end{aligned}$$

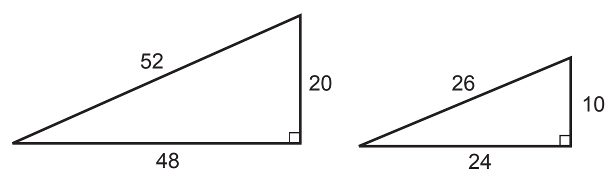
78.
$$\begin{aligned} m\angle EOD &= 180° - m\angle EOA - m\angle BOD \\ &= 180° - 85° - 15° \\ &= 80° \\ m\angle OED &= m\angle EDO \\ m\angle OED &= \tfrac{1}{2}(180° - m\angle EOD) \\ &= \tfrac{1}{2}(180° - 80°) \\ &= 50° \\ m\angle ECA &= 180° - m\angle EOC - m\angle OED \\ &= 180° - (80° + 15°) - 50° \\ &= 35° \end{aligned}$$

The correct choice is D.

5-2 Trigonometric Ratios in Right Triangles

Page 284 Graphing Calculator Exploration

1. Sample answers:



2. $R_1 = \frac{5}{13}$ or about 0.3846

$R_1 = \frac{15}{39}$ or about 0.3846

$R_2 = \frac{12}{13}$ or about 0.9231

$R_2 = \frac{36}{39}$ or about 0.9231

$R_3 = \frac{5}{12}$ or about 0.4167

$R_3 = \frac{15}{36}$ or about 0.4167

3. $R_1 = \frac{12}{13}$ or about 0.9231

$R_1 = \frac{36}{39}$ or about 0.9231

$R_2 = \frac{5}{13}$ or about 0.3846

$R_2 = \frac{15}{39}$ or about 0.3846

$R_3 = \frac{12}{5}$ or 2.4

$R_3 = \frac{36}{15}$ or 2.4

4. Each ratio has the same value for all 22.6° angles.

5. yes

6. Yes; the triangles are similar.

Pages 287–288 Check for Understanding

1. The side opposite the acute angle of a right triangle is the side that is not part of either side of the angle. The side adjacent to the acute angle is the side of the triangle that is part of the side of the angle, but is not the hypotenuse.
2. cosecant; secant; cotangent
3. $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$, $\csc A = \frac{c}{a}$, $\sec A = \frac{c}{b}$, $\cot A = \frac{b}{a}$
4. $\sin A = \cos B$, $\csc A = \sec B$, $\tan A = \cot B$
5. $(TV)^2 + (VU)^2 = (TU)^2$
   $17^2 + 15^2 = (TU)^2$
   $514 = (TU)^2$
   $\sqrt{514} = TU$
   $\sin T = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos T = \frac{\text{side adjacent}}{\text{hypotenuse}}$
   $\sin T = \frac{15}{\sqrt{514}}$ or $\frac{15\sqrt{514}}{514}$ $\cos T = \frac{17}{\sqrt{514}}$ or $\frac{17\sqrt{514}}{514}$
   $\tan T = \frac{\text{side opposite}}{\text{side adjacent}}$
   $\tan T = \frac{15}{17}$
6. $\csc \theta = \frac{1}{\sin \theta}$
   $\csc \theta = \frac{1}{\frac{2}{5}}$ or $\frac{5}{2}$
7. $\tan \theta = \frac{1}{\cot \theta}$
   $\tan \theta = \frac{1}{1.5}$ or about 0.6667
8. $(PS)^2 + (QS)^2 = (QP)^2$
   $(PS)^2 + 6^2 = 20^2$
   $(PS)^2 = 364$
   $PS = \sqrt{364}$ or $2\sqrt{91}$
   $\sin P = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos P = \frac{\text{side adjacent}}{\text{hypotenuse}}$
   $\sin P = \frac{6}{20}$ or $\frac{3}{10}$ $\cos P = \frac{2\sqrt{91}}{20}$ or $\frac{\sqrt{91}}{10}$
   $\tan P = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc P = \frac{\text{hypotenuse}}{\text{side opposite}}$
   $\tan P = \frac{6}{2\sqrt{91}}$ or $\frac{3\sqrt{91}}{91}$ $\csc P = \frac{20}{6}$ or $\frac{10}{3}$
   $\sec P = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot P = \frac{\text{side adjacent}}{\text{side opposite}}$
   $\sec P = \frac{20}{2\sqrt{91}}$ or $\frac{10\sqrt{91}}{91}$ $\cot P = \frac{2\sqrt{91}}{6}$ or $\frac{\sqrt{91}}{3}$
9. $\cos \theta = \sqrt{\frac{I_t}{I_o}}$
   $\cos 45° = \frac{\sqrt{I_t}}{I_o}$
   $\frac{\sqrt{2}}{2} = \frac{\sqrt{I_t}}{\sqrt{I_o}}$
   $\frac{2}{4} = \frac{I_t}{I_o}$
   $0.5I_0 = I_t$

Pages 288–290 Exercises

10. $(AC)^2 + (CB)^2 = (AB)^2$
    $80^2 + 60^2 = (AB)^2$
    $10{,}000 = (AB)^2$
    $100 = AB$
    $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
    $\sin A = \frac{60}{100}$ or $\frac{3}{5}$ $\cos A = \frac{80}{100}$ or $\frac{4}{5}$
    $\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$
    $\tan A = \frac{60}{80}$ or $\frac{3}{4}$
11. $(AC)^2 + (CB)^2 = (AB)^2$
    $8^2 + 5^2 = (AB)^2$
    $89 = (AB)^2$
    $\sqrt{89} = AB$
    $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
    $\sin A = \frac{5}{\sqrt{89}}$ or $\frac{5\sqrt{89}}{89}$ $\cos A = \frac{8}{\sqrt{89}}$ or $\frac{8\sqrt{89}}{89}$
    $\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$
    $\tan A = \frac{5}{8}$
12. $(AC)^2 + (BC)^2 = (AB)^2$
    $(AC)^2 + 12^2 = 40^2$
    $(AC)^2 = 1456$
    $AC = \sqrt{1456}$ or $4\sqrt{91}$
    $\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}}$
    $\sin A = \frac{12}{40}$ or $\frac{3}{10}$ $\cos A = \frac{4\sqrt{91}}{40}$ or $\frac{\sqrt{91}}{10}$
    $\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$
    $\tan A = \frac{12}{4\sqrt{91}}$ or $\frac{3\sqrt{91}}{91}$
13. tangent
14. $\cot \theta = \frac{1}{\tan \theta}$
    $\cot \theta = \frac{1}{\frac{1}{3}}$ or 3
15. $\csc \theta = \frac{1}{\sin \theta}$
    $\csc \theta = \frac{1}{\frac{3}{7}}$ or $\frac{7}{3}$
16. $\cos \theta = \frac{1}{\sec \theta}$
    $\cos \theta = \frac{1}{\frac{5}{9}}$ or $\frac{9}{5}$
17. $\sin \theta = \frac{1}{\csc \theta}$
    $\sin \theta = \frac{1}{2.5}$ or 0.4
18. $\tan \theta = \frac{1}{\cot \theta}$
    $\tan \theta = \frac{1}{0.75}$ or about 1.3333
19. $\sec \theta = \frac{1}{\cos \theta}$
    $\sec \theta = \frac{1}{0.125}$ or 8
20. $(RT)^2 + (TS)^2 = (RS)^2$
    $14^2 + (TS)^2 = 48^2$
    $(TS)^2 = 2108$
    $TS = \sqrt{2108}$ or $2\sqrt{527}$
    $\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$
    $\sin R = \frac{2\sqrt{527}}{48}$ or $\frac{\sqrt{527}}{24}$ $\cos R = \frac{14}{48}$ or $\frac{7}{24}$
    $\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$
    $\tan R = \frac{2\sqrt{527}}{14}$ or $\frac{\sqrt{527}}{7}$ $\csc R = \frac{48}{2\sqrt{527}}$ or $\frac{24\sqrt{527}}{527}$
    $\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$
    $\sec R = \frac{48}{14}$ or $\frac{24}{7}$ $\cot R = \frac{14}{2\sqrt{527}}$ or $\frac{7\sqrt{527}}{527}$

21. $(ST)^2 + (TR)^2 = (SR)^2$
$38^2 + (TR)^2 = 40^2$
$(TR)^2 = 156$
$TR = \sqrt{156}$ or $2\sqrt{39}$

$\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin R = \frac{38}{40}$ or $\frac{19}{20}$ $\quad$ $\cos R = \frac{2\sqrt{39}}{40}$ or $\frac{\sqrt{39}}{20}$

$\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan R = \frac{38}{2\sqrt{39}}$ or $\frac{19\sqrt{39}}{39}$ $\quad$ $\csc R = \frac{40}{38}$ or $\frac{20}{19}$

$\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec R = \frac{40}{2\sqrt{39}}$ or $\frac{20\sqrt{39}}{39}$ $\quad$ $\cot R = \frac{2\sqrt{39}}{38}$ or $\frac{\sqrt{39}}{19}$

22. $(ST)^2 + (TR)^2 = (SR)^2$
$(\sqrt{7})^2 - 9^2 = (SR)^2$
$88 = (SR)^2$
$\sqrt{88} = SR$; $\sqrt{88}$ or $2\sqrt{22}$

$\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin R = \frac{\sqrt{7}}{2\sqrt{22}}$ or $\frac{\sqrt{154}}{44}$ $\quad$ $\cos R = \frac{9}{2\sqrt{22}}$ or $\frac{9\sqrt{22}}{44}$

$\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan R = \frac{\sqrt{7}}{9}$ $\quad$ $\csc R = \frac{2\sqrt{22}}{\sqrt{7}}$ or $\frac{2\sqrt{154}}{7}$

$\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec R = \frac{2\sqrt{22}}{9}$ $\quad$ $\cot R = \frac{9}{\sqrt{7}}$ or $\frac{9\sqrt{7}}{7}$

23. $\cot (90° - \theta) = \tan \theta$
$\cot (90° - \theta) = 1.3$

24a. 0.7963540136

24b. 0.186524036

24c. 35.34015106

24d. 1.37638192

25.

θ	**72°**	**74°**	**76°**	**78°**	**80°**
sin	0.951	0.961	0.970	0.978	0.985
cos	0.309	0.276	0.242	0.208	0.174

θ	**82°**	**84°**	**86°**	**88°**
sin	0.990	0.995	0.998	0.999
cos	0.139	0.105	0.070	0.035

25a. 1

25b. 0

26.

θ	**18°**	**16°**	**14°**	**12°**	**10°**
sin	0.309	0.276	0.242	0.208	0.174
cos	0.951	0.961	0.970	0.978	0.985
tan	0.325	0.287	0.249	0.213	0.176

θ	**8°**	**6°**	**4°**	**2°**
sin	0.139	0.105	0.070	0.035
cos	0.990	0.995	0.998	0.999
tan	0.141	0.105	0.070	0.035

26a. 0

26b. 1

26c. 0

27. $\frac{\sin \theta_i}{\sin \theta_r} = n$
$\frac{\sin 45°}{\sin 27° 55'} = n$
$1.5103 \approx n$

28. $\sin R = \frac{\text{side opposite}}{\text{hypotenuse}}$
$\sin R = \frac{3}{7}$
$a^2 + b^2 = c^2$
$3^2 + b^2 = 7^2$
$b^2 = 40$
$b = \sqrt{40}$ or $2\sqrt{10}$

$\cos R = \frac{\text{side adjacent}}{\text{hypotenuse}}$ $\quad$ $\tan R = \frac{\text{side opposite}}{\text{side adjacent}}$

$\cos R = \frac{2\sqrt{10}}{7}$ $\quad$ $\tan R = \frac{3}{2\sqrt{10}}$ or $\frac{3\sqrt{10}}{20}$

$\csc R = \frac{\text{hypotenuse}}{\text{side opposite}}$ $\quad$ $\sec R = \frac{\text{hypotenuse}}{\text{side adjacent}}$

$\csc R = \frac{7}{3}$ $\quad$ $\sec R = \frac{7}{2\sqrt{10}}$ or $\frac{7\sqrt{10}}{20}$

$\cot R = \frac{\text{side adjacent}}{\text{side opposite}}$

$\cot R = \frac{2\sqrt{10}}{3}$

29a. $\tan \theta = \frac{v^2}{gr}$
$\tan 11° = \frac{v^2}{9.8(15.5)}$
$29.53 \approx v^2$
$5.4 \approx v$
about 5.4 m/s

29b. $\tan \theta = \frac{v^2}{gr}$
$\tan 13° = \frac{v^2}{9.8(15.5)}$
$35.07 \approx v^2$
$5.9 \approx v$
about 5.9 m/s

29c. $\tan \theta = \frac{v^2}{gr}$
$\tan 15° = \frac{v^2}{9.8(15.5)}$
$40.70 \approx v^2$
$6.4 \approx v$
about 6.4 m/s

29d. increase

30. $\sin \theta = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos \theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{\text{side opposite}}{\text{hypotenuse}}}{\frac{\text{side adjacent}}{\text{hypotenuse}}}$$
$$\frac{\sin \theta}{\cos \theta} = \frac{\text{side opposite}}{\text{hypotenuse}} \div \frac{\text{side adjacent}}{\text{hypotenuse}}$$
$$\frac{\sin \theta}{\cos \theta} = \frac{\text{side opposite}}{\text{hypotenuse}} \cdot \frac{\text{hypotenuse}}{\text{side adjacent}}$$
$$\frac{\sin \theta}{\cos \theta} = \frac{\text{side opposite}}{\text{side adjacent}}$$
$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

31a. $\angle = 90° - L - 23.5° \times \cos\left[\frac{(N + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times \cos\left[\frac{(172 + 10)360}{365}\right]$
$\angle \approx 90° - 26° - 23.5° \times (-0.99997)$
$\angle \approx 87.5°$
$\angle = 90° - L - 23.5° \times \cos\left[\frac{(N + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times \cos\left[\frac{(355 + 10)360}{365}\right]$
$\angle = 90° - 26° - 23.5° \times 1$
$\angle = 40.5°$

31b. $\angle = 90° - L - 23.5° \times \cos\left[\frac{(N + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times \cos\left[\frac{(172 + 10)360}{365}\right]$
$\angle \approx 90° - 64° - 23.5° \times -0.99997$
$\angle \approx 49.5°$
$\angle = 90° - L - 23.5° \times \cos\left[\frac{(N + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times \cos\left[\frac{(355 + 10)360}{365}\right]$
$\angle = 90° - 64° - 23.5° \times 1$
$\angle = 2.5°$

31c. $87.5° - 40.5° = 47°$
$49.5° - 25° = 47°$
neither

32. $x = t\left(\frac{\sin(B - A)}{\cos A}\right)$

$x = 10\left(\frac{\sin(60° - 41°)}{\cos 41°}\right)$

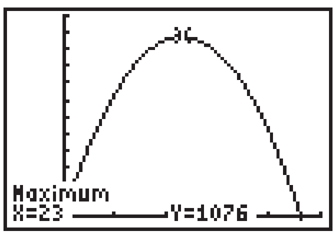
$x \approx 10(0.4314)$

$x \approx 4.31$; about 4.31 cm

33. $88.37° = 88° + (0.37 \cdot 60)'$
$= 88° + 22.2'$
$= 88° + 22' + (0.2 \cdot 60)''$
$= 88° + 22' + 12''$
$88°\ 22'\ 12''$

34. positive: 1
$f(-x) = x^4 - 2x^3 + 6x - 1$
negative: 3 or 1

35.



35a. 23 employees
35b. \$1076

$[-10, 50]$ scl:10 by $[-10, 1200]$ scl:100

36. $\begin{vmatrix} 7 & -3 & 5 \\ 4 & 0 & -1 \\ 8 & 2 & 0 \end{vmatrix} = 7\begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -1 \\ 8 & 0 \end{vmatrix} + 5\begin{vmatrix} 4 & 0 \\ 8 & 2 \end{vmatrix}$

$= 7(2) - (-3)(8) + 5(8)$
$= 78$

37. $m = \frac{3 - 5}{6 - 2}$

$m = \frac{-2}{4}$ or $-\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{2}(x - 6)$

$y = -\frac{1}{2}x + 6$

38. $A = \frac{1}{2}bh$

$12 = \frac{1}{2}(2x)(3x)$

$12 = 3x^2$
$4 = x^2$
$2 = x$

$2x = 2(2)$ or 4

$3x = 3(2)$ or 6

$a^2 + b^2 = c^2$
$4^2 + 6^2 = c^2$
$52 = c^2$
$\sqrt{52} = c$; $\sqrt{52}$ or $2\sqrt{13}$

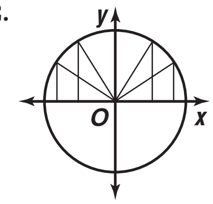
The correct choice is C.

5-3 Trigonometric Functions on the Unit Circle

Page 296 Check for Understanding

1. Terminal side of a 180° angle in standard position is the negative x-axis which intersects the unit circle at $(-1, 0)$. Since $\csc \theta = \frac{1}{y}$, $\csc 180° = \frac{1}{0}$ which is undefined.

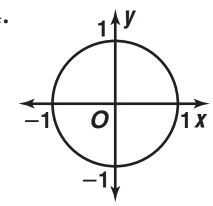
2.



As θ goes from 0° to 90°, the y-coordinate increases. As θ goes from 90° to 180°, the y-coordinate decreases.

3. $\cot \theta = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$

4.



Function	Quadrant			
	I	II	III	IV
sin ∝ or cos ∝	+	+	−	−
cos ∝ or sec ∝	+	−	−	+
tan ∝ or cot ∝	+	−	+	−

5. $(-1, 0)$; $\tan 180° = \frac{y}{x}$ or $\frac{0}{-1}$; 0

6. $(0, -1)$; $\sec(-90°) = \frac{1}{x}$ or $\frac{1}{0}$; undefined

7. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 30° = y$ $\qquad$ $\cos 30° = x$

$\sin 30° = \frac{1}{2}$ $\qquad$ $\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{y}{x}$ $\qquad$ $\csc 30° = \frac{1}{y}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\qquad$ $\csc 30° = \frac{1}{\frac{1}{2}}$

$\csc 30° = 2$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan 30° = \frac{\sqrt{3}}{3}$

$\sec 30° = \frac{1}{x}$ $\qquad$ $\cot 30° = \frac{x}{y}$

$\sec 30° = \frac{1}{\frac{\sqrt{3}}{2}}$ $\qquad$ $\cot 30° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\sec 30° = \frac{2}{\sqrt{3}}$ $\qquad$ $\cot 30° = \sqrt{3}$

$\sec 30° = \frac{2\sqrt{3}}{3}$

8. terminal side — Quadrant III
reference angle: 225° − 180° = 45°
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 225° = y$ $\quad$ $\cos 225° = x$

$\sin 225° = -\frac{\sqrt{2}}{2}$ $\quad$ $\cos 225° = -\frac{\sqrt{2}}{2}$

$\tan 225° = \frac{y}{x}$ $\quad$ $\csc 225° = \frac{1}{y}$

$\tan 225° = \dfrac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ $\quad$ $\csc 225° = \dfrac{1}{-\frac{\sqrt{2}}{2}}$

$\tan 225° = 1$ $\quad$ $\csc 225° = -\frac{2}{\sqrt{2}}$

$\csc 225° = -\sqrt{2}$

$\sec 225° = \frac{1}{x}$ $\quad$ $\cot 225° = \frac{x}{y}$

$\sec 225° = \dfrac{1}{-\frac{\sqrt{2}}{2}}$ $\quad$ $\cot 225° = \dfrac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$\sec 225° = -\frac{2}{\sqrt{2}}$ $\quad$ $\cot 225° = 1$

$\sec 225° = -\sqrt{2}$

9. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{3^2 + 4^2}$
$t = \sqrt{25}$ or 5

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{4}{5}$ $\quad$ $\cos\theta = \frac{3}{5}$ $\quad$ $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{5}{4}$ $\quad$ $\sec\theta = \frac{5}{3}$ $\quad$ $\cot\theta = \frac{3}{4}$

10. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-6)^2 + 6^2}$
$r = \sqrt{72}$ or $6\sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{6}{6\sqrt{2}}$ $\quad$ $\cos\theta = \frac{-6}{6\sqrt{2}}$ $\quad$ $\tan\theta = \frac{6}{-6}$ or -1

$\sin\theta = \frac{\sqrt{2}}{2}$ $\quad$ $\cos\theta = -\frac{\sqrt{2}}{2}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{6\sqrt{2}}{6}$ or $\sqrt{2}$ $\quad$ $\sec\theta = \frac{6\sqrt{2}}{-6}$ or $-\sqrt{2}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{-6}{6}$ or -1

11. $\tan\theta = \frac{y}{x}$ $\quad$ $r^2 = x^2 + y^2$
$\tan\theta = -1$ $\quad$ $r^2 = 1^2 + (-1)^2$
$x = 1, y = -1$ $\quad$ $r^2 = 2$
$r = \sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ $\quad$ $\cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\quad$ $\cos\theta = \frac{\sqrt{2}}{2}$

$\csc\theta = \frac{r}{y}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\csc\theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$ $\quad$ $\sec\theta = \frac{\sqrt{2}}{1}$ or $\sqrt{2}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{1}{-1}$ or -1

12. $\cos\theta = \frac{x}{r}$ $\quad$ $r^2 = x^2 + y^2$
$\cos\theta = -\frac{1}{2}$ $\quad$ $2^2 = (-1)^2 + y^2$
$x = -1, r = 2$ $\quad$ $3 = y^2$
$\pm\sqrt{3} = y$
Quadrant II, so y = $\sqrt{3}$

$\sin\theta = \frac{y}{r}$ $\quad$ $\tan\theta = \frac{y}{x}$ $\quad$ $\csc\theta = \frac{r}{y}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad$ $\tan\theta = \frac{\sqrt{3}}{-1}$ or $-\sqrt{3}$ $\quad$ $\csc\theta = \frac{2}{\sqrt{3}}$

$\csc\theta = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{r}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{2}{-1}$ or -2 $\quad$ $\cot\theta = \frac{-1}{\sqrt{3}}$

$\cot\theta = \frac{-\sqrt{3}}{3}$

13. $C = 2\pi r \cos L$ $\quad$ $C = 2\pi r \cos L$
$C = 2\pi(3960) \cos 0°$ $\quad$ $C = 2\pi(3960) \cos 90°$
$C \approx 24{,}881.41$ $\quad$ $C = 0$
The circumference goes from about 24,881 miles to 0 miles.

Pages 296–298 Exercises

14. (0, 1); $\sin 90° = y$ or 1

15. (1, 0); $\tan 360° = \frac{y}{x}$ or $\frac{0}{1}$; 0

16. (−1, 0); $\cot(-180°) = \frac{x}{y}$ or $\frac{-1}{0}$; undefined

17. (0, −1); $\csc 270° = \frac{1}{y}$ or $\frac{1}{-1}$; −1

18. (0, 1); $\cos(-270°) = x$ or 0

19. (−1, 0); $\sec 180° = \frac{1}{x}$ or $\frac{1}{-1}$; −1

20. Sample answers: 0°, 180° $\quad$ 21. undefined

22. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin 45° = y$ $\quad$ $\cos 45° = x$

$\sin 45° = \frac{\sqrt{2}}{2}$ $\quad$ $\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{y}{x}$ $\quad$ $\csc 45° = \frac{1}{y}$

$\tan 45° = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1 $\quad$ $\csc 45° = \dfrac{1}{\frac{\sqrt{2}}{2}}$

$\csc 45° = \frac{2}{\sqrt{2}}$

$\csc 45° = \sqrt{2}$

$\sec 45° = \frac{1}{x}$ $\quad$ $\cot 45° = \frac{x}{y}$

$\sec 45° = \dfrac{1}{\frac{\sqrt{2}}{2}}$ $\quad$ $\cot 45° = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1

$\sec 45° = \frac{2}{\sqrt{2}}$

$\sec 45° = \sqrt{2}$

23. terminal side — Quadrant II
reference angle: $180° - 150° = 30°$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 150° = y$ $\cos 150° = x$

$\sin 150° = \frac{1}{2}$ $\cos 150° = -\frac{\sqrt{3}}{2}$

$\tan 150° = \frac{y}{x}$ $\csc 150° = \frac{1}{y}$

$\tan 150° = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ $\csc 150° = \frac{1}{\frac{1}{2}}$

$\tan 150° = -\frac{1}{\sqrt{3}}$ $\csc 150° = 2$

$\tan 150° = -\frac{\sqrt{3}}{3}$

$\sec 150° = \frac{1}{x}$ $\cot 150° = \frac{x}{y}$

$\sec 150° = \frac{1}{-\frac{\sqrt{3}}{2}}$ $\cot 150° = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\sec 150° = -\frac{2}{\sqrt{3}}$ $\cot 150° = -\sqrt{3}$

$\sec 150° = -\frac{2\sqrt{3}}{3}$

24. terminal side — Quadrant IV
reference angle: $360° - 315° = 45°$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 315° = y$ $\cos 315° = x$

$\sin 315° = -\frac{\sqrt{2}}{2}$ $\cos 315° = \frac{\sqrt{2}}{2}$

$\tan 315° = \frac{y}{x}$ $\csc 315° = \frac{1}{y}$

$\tan 315° = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1 $\csc 315° = \frac{1}{-\frac{\sqrt{2}}{2}}$

$\csc 315° = -\frac{2}{\sqrt{2}}$

$\csc 315° = -\sqrt{2}$

$\sec 315° = \frac{1}{x}$ $\cot 315° = \frac{x}{y}$

$\sec 315° = \frac{1}{\frac{\sqrt{2}}{2}}$ $\cot 315° = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ or -1

$\sec 315° = \frac{2}{\sqrt{2}}$

$\sec 315° = \sqrt{2}$

25. terminal side — Quadrant III
reference angle: $210° - 180° = 30°$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin 210° = y$ $\cos 210° = x$

$\sin 210° = -\frac{1}{2}$ $\cos 210° = -\frac{\sqrt{3}}{2}$

$\tan 210° = \frac{y}{x}$ $\csc 210° = \frac{1}{y}$

$\tan 210° = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ $\csc 210° = \frac{1}{-\frac{1}{2}}$

$\tan 210° = \frac{1}{\sqrt{3}}$ $\csc 210° = -2$

$\tan 210° = \frac{\sqrt{3}}{3}$

$\sec 210° = \frac{1}{x}$ $\cot 210° = \frac{x}{y}$

$\sec 210° = \frac{1}{-\frac{\sqrt{3}}{2}}$ $\cot 210° = -\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\sec 210° = -\frac{2}{\sqrt{3}}$ $\cot 210° = \sqrt{3}$

$\sec 210° = -\frac{2\sqrt{3}}{3}$

26. terminal side — Quadrant IV
reference angle: $360° - 330° = 30°$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sin 330° = y$ $\cos 330° = x$

$\sin 330° = -\frac{1}{2}$ $\cos 330° = \frac{\sqrt{3}}{2}$

$\tan 330° = \frac{y}{x}$ $\csc 330° = \frac{1}{y}$

$\tan 330° = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\csc 330° = \frac{1}{-\frac{1}{2}}$

$\tan 330° = -\frac{1}{\sqrt{3}}$ $\csc 330° = -2$

$\tan 330° = -\frac{\sqrt{3}}{3}$

$\sec 330° = \frac{1}{x}$ $\cot 330° = \frac{x}{y}$

$\sec 330° = \frac{1}{\frac{\sqrt{3}}{2}}$ $\cot 330° = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\sec 330° = \frac{2}{\sqrt{3}}$ $\cot 330° = -\sqrt{3}$

$\sec 330° = \frac{2\sqrt{3}}{3}$

27. terminal side — Quadrant I
reference angle: $420° - 360° = 60°$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin 420° = y$
$\sin 420° = \frac{\sqrt{3}}{2}$
$\tan 420° = \frac{y}{x}$
$\tan 420° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$
$\tan 420° = \sqrt{3}$
$\sec 420° = \frac{1}{x}$
$\sec 420° = \frac{1}{\frac{1}{2}}$
$\sec 420° = 2$

$\cos 420° = x$
$\cos 420° = \frac{1}{2}$
$\csc 420° = \frac{1}{y}$
$\csc 420° = \frac{1}{\frac{\sqrt{3}}{2}}$
$\csc 420° = \frac{2}{\sqrt{3}}$
$\csc 420° = \frac{2\sqrt{3}}{3}$
$\cot 420° = \frac{x}{y}$
$\cot 420° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$\cot 420° = \frac{1}{\sqrt{3}}$
$\cot 420° = \frac{\sqrt{3}}{3}$

28. terminal side — Quadrant IV
reference angle: 45°
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\cot(-45°) = \frac{x}{y}$
$\cot(-45°) = -\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1

29. terminal side — Quadrant 1
reference angle: $390° - 360° = 30°$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\csc 390° = \frac{1}{y}$
$\csc 390° = \frac{1}{\frac{1}{2}}$ or 2

30. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-4)^2 + (-3)^2}$
$r = \sqrt{25}$ or 5

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = -\frac{3}{5}$ $\quad\cos\theta = -\frac{4}{5}$ $\quad\tan\theta = \frac{-3}{-4}$ or $\frac{3}{4}$
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$
$\csc\theta = -\frac{5}{3}$ $\quad\sec\theta = -\frac{5}{4}$ $\quad\cot\theta = \frac{-4}{-3}$ or $\frac{4}{3}$

31. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-6)^2 + 6^2}$
$r = \sqrt{72}$ or $6\sqrt{2}$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{6}{6\sqrt{2}}$ $\quad\cos\theta = \frac{-6}{6\sqrt{2}}$ $\quad\tan\theta = \frac{6}{-6}$ or -1
$\sin\theta = \frac{\sqrt{2}}{2}$ $\quad\cos\theta = -\frac{\sqrt{2}}{2}$
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{6\sqrt{2}}{6}$ or $\sqrt{2}$ $\quad\sec\theta = \frac{6\sqrt{2}}{-6}$ or $-\sqrt{2}$ $\quad\cot\theta = \frac{-6}{6}$ or -1

32. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{2^2 + 0^2}$
$r = \sqrt{4}$ or 2

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{0}{2}$ or 0 $\quad\cos\theta = \frac{2}{2}$ or 1 $\quad\tan\theta = \frac{0}{2}$ or 0
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{2}{0}$ undefined $\quad\sec\theta = \frac{2}{2}$ or 1 $\quad\cot\theta = \frac{2}{0}$ undefined

33. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{1^2 + (-8)^2}$
$r = \sqrt{65}$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-8}{\sqrt{65}}$ $\quad\cos\theta = \frac{1}{\sqrt{65}}$ $\quad\tan\theta = \frac{-8}{1}$ or -8
$\sin\theta = -\frac{8\sqrt{65}}{65}$ $\quad\cos\theta = \frac{\sqrt{65}}{65}$
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$
$\csc\theta = \frac{\sqrt{65}}{-8}$ or $-\frac{\sqrt{65}}{8}$ $\quad\sec\theta = \frac{\sqrt{65}}{1}$ or $\sqrt{65}$
$\cot\theta = \frac{x}{y}$
$\cot\theta = \frac{1}{-8}$ or $-\frac{1}{8}$

34. $r = \sqrt{x^2 - y^2}$
$r = \sqrt{5^2 + (-3)^2}$
$r = \sqrt{34}$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-3}{\sqrt{34}}$ $\quad\cos\theta = \frac{5}{\sqrt{34}}$ $\quad\tan\theta = \frac{-3}{5}$ or $-\frac{3}{5}$
$\sin\theta = -\frac{3\sqrt{34}}{34}$ $\quad\cos\theta = \frac{5\sqrt{34}}{34}$
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$
$\csc\theta = \frac{\sqrt{34}}{-3}$ or $-\frac{\sqrt{34}}{3}$ $\quad\sec\theta = \frac{\sqrt{34}}{5}$
$\cot\theta = \frac{x}{y}$
$\cot\theta = \frac{5}{-3}$ or $-\frac{5}{3}$

35. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-8)^2 + 15^2}$
$r = \sqrt{289}$ or 17

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{15}{17}$ $\quad\cos\theta = \frac{-8}{17}$ or $-\frac{8}{17}$ $\quad\tan\theta = \frac{15}{-8}$ or $-\frac{15}{8}$
$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{17}{15}$ $\quad\sec\theta = \frac{17}{-8}$ or $-\frac{17}{8}$ $\quad\cot\theta = \frac{-8}{15}$ or $-\frac{8}{15}$

36. $r = \sqrt{x^2 + y^2}$ $r = \sqrt{x^2 + y^2}$
$r = \sqrt{5^2 + (-6)^2}$ $r = \sqrt{(-5)^2 + 6^2}$
$r = \sqrt{61}$ $r = \sqrt{61}$
$\sin\theta = \frac{y}{r}$ $\sin\theta = \frac{y}{r}$
$\sin\theta = \frac{-6}{\sqrt{61}}$ $\sin\theta = \frac{6}{\sqrt{61}}$
$\sin\theta = -\frac{6\sqrt{61}}{61}$ $\sin\theta = \frac{6\sqrt{61}}{61}$
The sine of one angle is the negative of the sine of the other angle.

37. If $\sin\theta < 0$, y must be negative, so the terminal side is located in Quadrant III or IV

38. $\cos\theta = \frac{x}{r}$ $r^2 = x^2 + y^2$
$\cos\theta = -\frac{12}{13}$ $13^2 = (-12)^2 + y^2$
$x = -12, r = 13$ $25 = y^2$
$\pm 5 = y$
Quadrant III, so $y = -5$
$\sin\theta = \frac{y}{r}$ $\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-5}{13}$ or $-\frac{5}{13}$ $\tan\theta = \frac{-5}{-12}$ or $\frac{5}{12}$
$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$
$\csc\theta = \frac{13}{-5}$ or $-\frac{13}{5}$ $\sec\theta = \frac{13}{-12}$ or $-\frac{13}{12}$
$\cot\theta = \frac{r}{y}$
$\cot\theta = \frac{-12}{-5}$ or $\frac{12}{5}$

39. $\csc\theta = \frac{r}{y}$ $r^2 = x^2 + y^2$
$\csc\theta = 2$ $2^2 = x^2 + 1^2$
$r = 2, y = 1$ $3 = x^2$
$\pm\sqrt{3} = x$
Quadrant II, so $x = -\sqrt{3}$
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{1}{2}$ $\cos\theta = \frac{-\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$ $\tan\theta = \frac{1}{-\sqrt{3}}$
$\tan\theta = -\frac{\sqrt{3}}{3}$
$\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$
$\sec\theta = \frac{2}{-\sqrt{3}}$ $\cot\theta = \frac{-\sqrt{3}}{1}$ or $-\sqrt{3}$
$\sec\theta = -\frac{2\sqrt{3}}{3}$

40. $\sin\theta = \frac{y}{r}$ $r^2 = x^2 + y^2$
$\sin\theta = -\frac{1}{5}$ $5^2 = x^2 + (-1)^2$
$y = -1, r = 5$ $24 = x^2$
$\pm 2\sqrt{6} = x$
Quadrant IV, so $x = 2\sqrt{6}$
$\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $\csc\theta = \frac{r}{y}$
$\cos\theta = \frac{2\sqrt{6}}{5}$ $\tan\theta = \frac{-1}{2\sqrt{6}}$ $\csc\theta = \frac{5}{-1}$ or -5
$\tan\theta = -\frac{\sqrt{6}}{12}$
$\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$
$\sec\theta = \frac{5}{2\sqrt{6}}$ $\cot\theta = \frac{2\sqrt{6}}{-1}$ or $-2\sqrt{6}$
$\sec\theta = \frac{5\sqrt{6}}{12}$

41. $\tan\theta = \frac{y}{x}$ $r^2 = x^2 + y^2$
$\tan\theta = 2$ $r^2 = 1^2 + 2^2$
$y = 2, x = 1$ $r^2 = 5$
$r = \sqrt{5}$
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\csc\theta = \frac{r}{y}$
$\sin\theta = \frac{2}{\sqrt{5}}$ $\cos\theta = \frac{1}{\sqrt{5}}$ $\csc\theta = \frac{\sqrt{5}}{2}$
$\sin\theta = \frac{2\sqrt{5}}{5}$ $\cos\theta = \frac{\sqrt{5}}{5}$
$\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$
$\sec\theta = \frac{\sqrt{5}}{1}$ or $\sqrt{5}$ $\cot\theta = \frac{1}{2}$

42. $\sec\theta = \frac{r}{x}$ $r^2 = x^2 + y^2$
$\sec\theta = \sqrt{3}$ $(\sqrt{3})^2 = 1^2 + y^2$
$r = \sqrt{3}, x = 1$ $2 = y^2$
$\pm\sqrt{2} = y$
Quadrant IV, so $y = -\sqrt{2}$
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-\sqrt{2}}{\sqrt{3}}$ $\cos\theta = \frac{1}{\sqrt{3}}$ $\tan\theta = \frac{-\sqrt{2}}{1}$ or $-\sqrt{2}$
$\sin\theta = -\frac{\sqrt{6}}{3}$ $\cos\theta = \frac{\sqrt{3}}{3}$
$\csc\theta = \frac{r}{y}$ $\cot\theta = \frac{x}{y}$
$\csc\theta = \frac{\sqrt{3}}{-\sqrt{2}}$ $\cot\theta = \frac{1}{-\sqrt{2}}$
$\csc\theta = -\frac{\sqrt{6}}{2}$ $\cot\theta = -\frac{\sqrt{2}}{2}$

43. $\cot\theta = \frac{x}{y}$ $r^2 = x^2 + y^2$
$\cot\theta = 1$ (Quadrant III) $r^2 = (-1)^2 + (-1)^2$
$x = -1, y = -1$ $r^2 = 2$
$r = \sqrt{2}$
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$
$\sin\theta = \frac{-1}{\sqrt{2}}$ $\cos\theta = \frac{-1}{\sqrt{2}}$ $\tan\theta = \frac{-1}{-1}$ or 1
$\sin\theta = -\frac{\sqrt{2}}{2}$ $\cos\theta = -\frac{\sqrt{2}}{2}$
$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$
$\csc\theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$ $\sec\theta = \frac{\sqrt{2}}{-1}$ or $-\sqrt{2}$

44. $\csc\theta = \frac{r}{y}$ $r^2 = x^2 + y^2$
$\csc\theta = -2$ $2^2 = x^2 + (-1)^2$
$r = 2, y = -1$ $3 = x^2$
$\pm\sqrt{3} = x$
Quadrant III, so $x = -\sqrt{3}$
$\tan\theta = \frac{y}{x}$
$\tan\theta = \frac{-1}{-\sqrt{3}}$
$\tan\theta = \frac{\sqrt{3}}{3}$

45. $g \sin\theta \cos\theta = 0$
$\sin\theta = 0$ or $\cos\theta = 0$
$\theta = 0°$ $\theta = 90°$

46a. k is an even integer. **46b.** k is an odd integer.

47. $\cos\theta = \sqrt{\frac{I_t}{I_o}}$
$\cos\theta = \sqrt{1}$ $I_t = I_o$
$\cos\theta = 1$
$\theta = 0°$

48. Let $x = -1$. $y = -3(-1)$

$$y = 3$$
$$r^2 = x^2 + y^2$$
$$r^2 = (-1)^2 + (3)^2$$
$$r^2 = 10$$
$$r = \sqrt{10}$$

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{3}{\sqrt{10}}$ $\quad\cos\theta = \frac{-1}{\sqrt{10}}$ $\quad\tan\theta = \frac{3}{-1}$ or -3

$\sin\theta = \frac{3\sqrt{10}}{10}$ $\quad\cos\theta = -\frac{\sqrt{10}}{10}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$

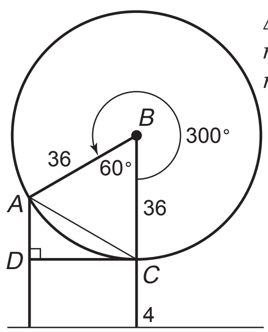
$\csc\theta = \frac{\sqrt{10}}{3}$ $\quad\sec\theta = \frac{\sqrt{10}}{-1}$ or $-\sqrt{10}$

$\cot\theta = \frac{x}{y}$

$\cot\theta = \frac{-1}{3}$ or $-\frac{1}{3}$

49a. $4 + 2(36) = 76$ ft

49b.



$\triangle ABC$ is equilateral.
$m\angle BCA = 60°$
$m\angle ACD + m\angle BCA = 90°$
$m\angle ACD + 60° = 90°$
$m\angle ACD = 30°$

Since $AC = 36$, $AD = 18$.
$18 + 4 = 22$ ft

49c. Refer to 49b for diagram and reasoning.
Since $AC = 30$, $AD = 15$.
$15 + 4 = 19$ ft

49d. $\frac{1}{2}r + 4$

50. $\sin\theta = \frac{1}{\csc\theta}$

$\sin\theta = \frac{1}{\frac{7}{5}}$

$\sin\theta = \frac{5}{7}$

51. $\frac{-840}{360} \approx -2.33$ $\qquad \alpha + 360(-2)° = -840°$

$$\alpha - 720° = -840°$$
$$\alpha = -120°$$

$360° - 120° = 240°$; III

52. $5 - \sqrt{b + 2} = 0$

$$5 = \sqrt{b+2}$$
$$25 = b + 2$$
$$23 = b$$

53. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{9 \pm \sqrt{(-9)^2 - 4(4)(5)}}{2(4)}$

$x = \frac{9 \pm \sqrt{1}}{8}$

$x = \frac{9+1}{8}$ or $x = \frac{9-1}{8}$

$x = \frac{10}{8}$ or 1.25 $\quad x = \frac{8}{8}$ or 1

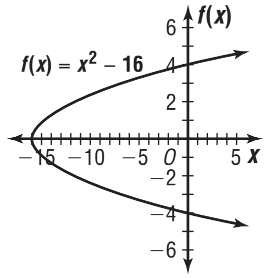
54. $k = \frac{y}{x}$ $\qquad y = kx$

$k = \frac{9}{-15}$ $\qquad y = (-0.6)(21)$

$k = -0.6$ $\qquad y = -12.6$

55.

$$f(x) = x^2 - 16$$
$$y = x^2 - 16$$
$$x = y^2 - 16$$
$$x + 16 = y^2$$
$$\pm\sqrt{x + 16} = y$$



56. $\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = 2(2) - (-3)1$

$$= 7$$

$\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

57. $3(8m - 3n - 4p) = 3(6)$ → $24m - 9n - 12p = 18$
$4m + 9n - 2p = -4$ → $4m + 9n - 2p = -4$
$28m - 14p = 14$

$4(8m - 3n - 4p) = 4(6)$ → $32m - 12n - 16p = 24$
$6m + 12n + 5p = -1$ → $6m + 12n + 5p = -1$
$38m - 11p = 23$

$11(28m - 14p) = 11(14)$
$-14(38m - 11p) = -14(23)$
↓
$308m - 154p = 154$
$-532m + 154p = -322$
$-224m = -168$

$$m = \frac{3}{4}$$

$38m - 11p = 23$ $\qquad 4m + 9n - 2p = -4$

$38\left(\frac{3}{4}\right) - 11p = 23$ $\qquad 4\left(\frac{3}{4}\right) + 9n - 2\left(\frac{1}{2}\right) = -4$

$p = \frac{1}{2}$ $\qquad n = -\frac{2}{3}$

$\left(\frac{3}{4}, -\frac{2}{3}, \frac{1}{2}\right)$

58. $2x - 4y \overset{?}{\le} 7$ $\qquad 2x - 4y \overset{?}{\le} 7$

$2(9) - 4(3) \overset{?}{\le} 7$ $\qquad 2(-1) - 4(2) \overset{?}{\le} 7$

$6 \le 7$; yes $\qquad -10 \le 7$; yes

$2x - 4y \overset{?}{\le} 7$

$2(2) - 4(-2) \overset{?}{\le} 7$

$12 \not\le 7$; no

59. absolute value; $f(x) = \left|2\frac{1}{2} - x\right|$

60. A of square $-$ A of circle $= A$

$$s^2 - \pi r^2 = A$$
$$2^2 - \pi(1)^2 = A$$
$$0.86 \approx A$$

The correct choice is C.

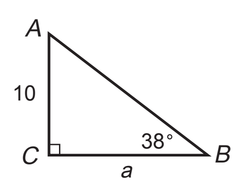
5-4 Applying Trigonometric Functions

Pages 301–302 Check for Understanding

1a. cos or sec $\quad$ **1b.** tan or cot

1c. sin or csc

2. Sample answer: Find a.



3. $\angle DCB$; $\angle ABC$; the measures are equal; if parallel lines are cut by a transversal, the alternate interior angles are congruent.

4. Sample answer: If you know the angle of elevation of the sun at noon on a particular day, you can measure the length of the shadow of the building at noon on that day. The height of the building equals the length of the shadow times the tangent of the angle of elevation of the sun.

5.
$$\tan A = \frac{a}{b}$$
$$\tan 76° = \frac{a}{113}$$
$$13 \tan 76° = a$$
$$52.1 \approx a$$

6.
$$\sin B = \frac{b}{c}$$
$$\sin 26° = \frac{18}{c}$$
$$c \sin 26° = 18$$
$$c = \frac{18}{\sin 26°}$$
$$c \approx 41.1$$

7.
$$\cos B = \frac{a}{c}$$
$$\cos 16° 45' = \frac{a}{13}$$
$$13 \cos 16° 45' = a$$
$$12.4 \approx a$$

8a. Let x = altitude.
$$\sin 55° 30' = \frac{x}{10}$$
$$10 \sin 55° 30' = x$$
$$8.2 \approx x$$
about 8.2 cm

8b. Let $x = \frac{1}{2}$ of the base.
$$\cos 55° 30' = \frac{x}{10}$$
$$10 \cos 55° 30' = x$$
$$5.66 \approx x$$
base = $2x$
base $\approx 2(5.66)$
base $\approx$ 11.3 cm

8c.
$$A = \frac{1}{2}bh$$
$$A \approx \frac{1}{2}(11.3)(8.2)$$
$$A \approx 46.7 \text{ cm}^2$$

9.
$$\tan 13° 15' = \frac{175}{x}$$
$$x \tan 13° 15' = 175$$
$$x = \frac{175}{\tan 13° 15'}$$
$$x \approx 743.2 \text{ ft}$$

Pages 302–304 Exercises

10.
$$\tan A = \frac{a}{b}$$
$$\tan 37° = \frac{a}{6}$$
$$6 \tan 37° = a$$
$$4.5 \approx a$$

11.
$$\cos B = \frac{a}{c}$$
$$\cos 67° = \frac{a}{16}$$
$$16 \cos 67° = a$$
$$6.3 \approx a$$

12.
$$\sin B = \frac{b}{c}$$
$$\sin 62° = \frac{b}{24}$$
$$24 \sin 62° = b$$
$$21.2 \approx b$$

13.
$$\sin A = \frac{a}{c}$$
$$\sin 29° = \frac{4.6}{c}$$
$$c \sin 29° = 4.6$$
$$c = \frac{4.6}{\sin 29°}$$
$$c \approx 9.5$$

14.
$$\cos B = \frac{a}{c}$$
$$\cos 77° = \frac{17.3}{c}$$
$$c \cos 77° = 17.3$$
$$c = \frac{17.3}{\cos 77°}$$
$$c \approx 76.9$$

15.
$$\tan B = \frac{b}{a}$$
$$\tan 61° = \frac{33.2}{a}$$
$$a \tan 61° = 33.2$$
$$a = \frac{33.2}{\tan 61°}$$
$$a \approx 18.4$$

16.
$$\tan B = \frac{b}{a}$$
$$\tan 49° 13' = \frac{10}{a}$$
$$a \tan 49° 13' = 10$$
$$a = \frac{10}{\tan 49° 13'}$$
$$a \approx 8.6$$

17.
$$\sin A = \frac{a}{c}$$
$$\sin 16° 55' = \frac{a}{13.7}$$
$$13.7 \sin 16° 55' = a$$
$$4.0 \approx a$$

18.
$$\cos B = \frac{a}{c}$$
$$\cos 47° 18' = \frac{22.3}{c}$$
$$c \cos 47° 18' = 22.3$$
$$c = \frac{22.3}{\cos 47° 18'}$$
$$c \approx 32.9$$

19.
$$\sin 30° = \frac{h}{12}$$
$$12 \sin 30° = h$$
$$6 = h$$
$$\tan 45° = \frac{6}{m}$$
$$m \tan 45° = 6$$
$$m = \frac{6}{\tan 45°}$$
$$m = 6$$

$$\cos 30° = \frac{n}{12}$$
$$12 \cos 30° = n$$
$$10.4 \approx n$$
$$\sin 45° = \frac{6}{p}$$
$$p \sin 45° = 6$$
$$p = \frac{6}{\sin 45°}$$
$$p \approx 8.5$$

20a.
$$\cos 36° = \frac{10.8}{x}$$
$$x \cos 36° = 10.8$$
$$x = \frac{10.8}{\cos 36°}$$
$$x \approx 13.3 \text{ cm}$$

20b.
$$\tan 36° = \frac{\frac{1}{2}s}{10.8}$$
$$10.8 \tan 36° = \frac{1}{2} s$$
$$2 \cdot 10.8 \tan 36° = s$$
$$15.7 \approx s$$
about 15.7 cm

20c. $P = 5s$
$P \approx 5(15.7)$
$P \approx 78.5$ cm

21a.
$$\cos 42° 30' = \frac{\frac{1}{2}(14.6)}{x}$$
$$x \cos 42° 30' = \frac{1}{2}(14.6)$$
$$x = \frac{\frac{1}{2}(14.6)}{\cos 42° 30'}$$
$$x \approx 9.9 \text{ m}$$

21b.
$$\tan 42° 30' = \frac{x}{\frac{1}{2}(14.6)}$$
$$\frac{1}{2}(14.6) \tan 42° 30' = x$$
$$6.7 \approx x$$
about 6.7 m

21c. $A = \frac{1}{2}bh$

$A \approx \frac{1}{2}(14.6)(6.7)$

$A \approx 48.8 \text{ m}^2$

22a. $r = \frac{1}{2}(6.4)$ or 3.2

$\cos 30° = \frac{a}{3.2}$

$3.2 \cos 30° = a$

$2.771281292 = a$

about 2.8 cm

22b. Let x = side of hexagon.

$\sin 30° = \frac{\frac{1}{2}x}{3.2}$

$32 \sin 30° = \frac{1}{2}x$

$2 \cdot 3.2 \sin 30° = x$

$3.2 = x$; 32 cm

22c. $P = 6s$

$P = 6(3.2)$

$P = 19.2$ cm

22d. $A = \frac{1}{2}pa$

$A \approx \frac{1}{2}(19.2)(2.771281292)$

$A \approx 26.6 \text{ cm}^2$

23. $\sin 10° \, 21' \, 36'' = \frac{195.8}{x}$

$x \sin 10° \, 21' \, 36'' = 195.8$

$x = \frac{195.8}{\sin 10° \, 21' \, 36''}$

$x \approx 1088.8$ ft

24. height:

$\tan \propto = \frac{x}{\frac{1}{2}s}$

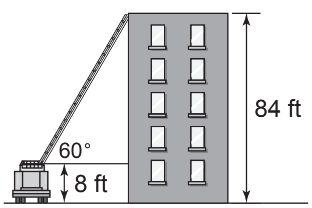
$\frac{1}{2}s \tan \propto = x$

$V = \frac{1}{3}$ area of base · height

$V = \frac{1}{3}(s^2)\left(\frac{1}{2}s \tan \propto\right)$

$V = \frac{1}{6}s^3 \tan \propto$

25a.



25b. $84 - 8 = 76$

$\tan 60° = \frac{76}{x}$

$x \tan 60° = 76$

$x = \frac{76}{\tan 60°}$

$x \approx 43.9$ ft

25c. $\sin 60° = \frac{76}{x}$

$x \sin 60° = 76$

$x = \frac{76}{\sin 60°}$

$x \approx 87.8$ ft

26a. $\tan 6° = \frac{3900}{x}$

$x \tan 6° = 3900$

$x = \frac{3900}{\tan 6°}$

$x \approx 37{,}106.0$ ft

26b. $\sin 6° = \frac{3900}{x}$

$x \sin 6° = 3900$

$x = \frac{3900}{\sin 6°}$

$x \approx 37{,}310.4$ ft

27. Yacht:

$\tan 20° = \frac{208}{x}$

$x \tan 20° = 208$

$x = \frac{208}{\tan 20°}$

$x \approx 571.5$

Barge:

$\tan 12° \, 30' = \frac{208}{x}$

$x \tan 12° \, 30' = 208$

$x = \frac{208}{\tan 12° \, 30'}$

$x \approx 938.2$

$938.2 - 571.5 \approx 366.8$ ft; no

28. Let M represent the point of intersection of the altitude and $\overline{EF}$. Since $\triangle GEF$ is isosceles, the altitude bisects $\overline{EF}$. $\triangle EMG$ is a right triangle. Therefore, $\sin \theta = \frac{a}{s}$ or $s \sin \theta = a$ and $\tan \theta = \frac{a}{0.5b}$ or $0.5b \tan \theta = a$.

29. Latasha:

$\sin 35° = \frac{x}{250}$

$250 \sin 35° = x$

$143.4 \approx x$

Markisha:

$\sin 42° = \frac{x}{225}$

$225 \sin 42° = x$

$150.6 \approx x$

$1506 - 143.4 = 7.2$

Markisha's; about 7.2 ft

30. Let x = the height of the building.
Let y = the distance between the buildings.

$\tan 47° \, 30' = \frac{x}{y}$

$y \tan 47° \, 30' = x$

$y = \frac{x}{\tan 47° \, 30'}$

$\tan 54° \, 54' = \frac{40 + x}{y}$

$y \tan 54° \, 54' = 40 + x$

$y = \frac{40 + x}{\tan 54° \, 54'}$

$\frac{x}{\tan 47° \, 30'} = \frac{40 + x}{\tan 54° \, 54'}$

$\tan 54° \, 54'(x) = \tan 47° \, 30'(40 + x)$

$x \tan 54° \, 54' = 40 \tan 47° \, 30' + x \tan 47° \, 30'$

$x(\tan 54° \, 54' - \tan 47° \, 30') = 40 \tan 47° \, 30'$

$x = \frac{40 \tan 47° \, 30'}{\tan 54° \, 54' - \tan 47° \, 30'}$

$x \approx 131.7$ ft

31. terminal side — Quadrant II
reference angle: $180° - 120° = 60°$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin 120° = y$

$\sin 120° = \frac{\sqrt{3}}{2}$

$\tan 120° = \frac{y}{x}$

$\tan 120° = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\tan 120° = -\sqrt{3}$

$\sec 120° = \frac{1}{x}$

$\sec 120° = \frac{1}{-\frac{1}{2}}$

$\sec 120° = -2$

$\cos 120° = x$

$\cos 120° = -\frac{1}{2}$

$\csc 120° = \frac{1}{y}$

$\csc 120° = \frac{1}{\frac{\sqrt{3}}{2}}$

$\csc 120° = \frac{2\sqrt{3}}{3}$

$\cot 120° = \frac{x}{y}$

$\cot 120° = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\cot 120° = -\frac{\sqrt{3}}{3}$

32. $(PR)^2 + (RQ)^2 = (PQ)^2$

$7^2 + 2^2 = (PQ)^2$

$53 = (PQ)^2$

$\sqrt{53} = PQ$

$\sin P = \frac{p}{r}$

$\sin P = \frac{2}{\sqrt{53}}$

$\sin P = \frac{2\sqrt{53}}{53}$

$\tan P = \frac{p}{q}$

$\tan P = \frac{2}{7}$

$\cos P = \frac{q}{r}$

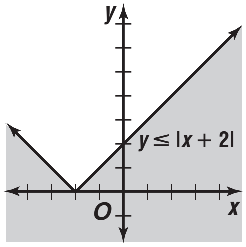
$\cos P = \frac{7}{\sqrt{53}}$

$\cos P = \frac{7\sqrt{53}}{53}$

33. $43° \, 15' \, 35'' = 43° + 15'\left(\frac{1°}{60'}\right) + 35''\left(\frac{1°}{3600''}\right)$

$\approx 43.260°$

34.



35. Let x = the cost of notebooks and y = the cost of pencils.

$3x + 2y = 5.89$
$4x + y = 6.20$

$$\begin{aligned} 3x + 2y &= 5.80 \\ -2(4x + y) &= -2(6.20) \end{aligned} \quad \rightarrow \quad \begin{aligned} 3x + 2y &= 5.80 \\ -8x - 2y &= -12.40 \\ \hline -5x &= -6.60 \\ x &= \$1.32 \end{aligned}$$

$$\begin{aligned} 4x + y &= 6.20 \\ 4(1.32) + y &= 6.20 \\ y &= \$0.92 \end{aligned}$$

36. $\frac{m \text{ miles}}{h \text{ hours}} \cdot x \text{ hours} = \frac{mx}{h} \text{ miles}$

The correct choice is E.

Page 304 Mid-Chapter Quiz

1. $$\begin{aligned} 34.605° &= 34° + (0.605 \cdot 60)' \\ &= 34° + 36.3' \\ &= 34° + 36' + (0.3 \cdot 60)'' \\ &= 34° + 36' + 18'' \end{aligned}$$

$34° \, 36' \, 18''$

2. $\frac{-400°}{360°} \approx -1.11$

$-1.11 + 1 = -0.11$
$-0.11 \times 360° = -40°$
$360° - 40° = 320°$; IV

3. $$\begin{aligned} (GI)^2 + (IH)^2 &= (GH)^2 \\ (GI)^2 + 10^2 &= 12^2 \\ (GI)^2 &= 44 \\ GI &= \sqrt{44} \text{ or } 2\sqrt{11} \end{aligned}$$

$\sin G = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos G = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin G = \frac{10}{12}$ or $\frac{5}{6}$ $\quad$ $\cos G = \frac{2\sqrt{11}}{12}$ or $\frac{\sqrt{11}}{6}$

$\tan G = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc G = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan G = \frac{10}{2\sqrt{11}}$ $\quad$ $\csc G = \frac{12}{10}$ or $\frac{6}{5}$

$\tan G = \frac{5\sqrt{11}}{11}$

$\sec G = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot G = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec G = \frac{12}{2\sqrt{11}}$ $\quad$ $\cot G = \frac{2\sqrt{11}}{10}$ or $\frac{\sqrt{11}}{5}$

$\sec G = \frac{6\sqrt{11}}{11}$

4. $$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ r &= \sqrt{2^2 + (-5)^2} \\ r &= \sqrt{29} \end{aligned}$$

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-5}{\sqrt{29}}$ $\quad$ $\cos \theta = \frac{2}{\sqrt{29}}$ $\quad$ $\tan \theta = \frac{-5}{2}$ or $-\frac{5}{2}$

$\sin \theta = -\frac{5\sqrt{29}}{29}$ $\quad$ $\cos \theta = \frac{2\sqrt{29}}{29}$

$\csc \theta = \frac{r}{y}$ $\quad$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{\sqrt{29}}{-5}$ or $-\frac{\sqrt{29}}{5}$ $\quad$ $\sec \theta = \frac{\sqrt{29}}{2}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{2}{-5}$ or $-\frac{2}{5}$

5. $$\begin{aligned} \tan 27.8° &= \frac{550}{x} \\ x \tan 27.8° &= 550 \\ x &= \frac{550}{\tan 27.8°} \\ x &\approx 1043.2 \text{ ft} \end{aligned}$$

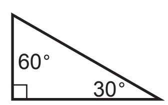
5-5 Solving Right Triangles

Pages 308–309 Check for Understanding

1a. linear $\quad$ **1b.** angle

2. They are complementary.

3. Sample answer:



4. Marta; they need to find the inverse of the cosine, not $\frac{1}{\cos}$.

5. 60°, 300° $\quad$ **6.** 150°, 330°

7. $\sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right)$

Let $A = \sin^{-1} \frac{\sqrt{3}}{2}$. Then $\sin A = \frac{\sqrt{3}}{2}$.

$\sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

8. Let $A = \cos^{-1} \frac{3}{5}$. Then $\cos A = \frac{3}{5}$

$$\begin{aligned} r^2 &= x^2 + y^2 \\ 5^2 &= 3^2 + y^2 \\ 16 &= y^2 \\ 4 &= y \end{aligned}$$

$\tan A = \frac{4}{3}$; $\tan\left(\cos^{-1} \frac{3}{5}\right) = \frac{4}{3}$

9. $\tan R = \frac{r}{s}$

$\tan R = \frac{7}{10}$

$R = \tan^{-1} \frac{7}{10}$

$R \approx 35.0°$

10. $\cos S = \frac{r}{t}$

$\cos S = \frac{12}{20}$

$S = \cos^{-1} \frac{12}{20}$

$S \approx 53.1°$

11. $A + 78° = 90°$

$A = 12°$

Find b.

$\tan B = \frac{b}{a}$

$\tan 78° = \frac{b}{41}$

$41 \tan 78° = b$

$192.9 \approx b$

Find c.

$\cos B = \frac{a}{c}$

$\cos 78° = \frac{41}{c}$

$c \cos 78° = 41$

$c = \frac{41}{\cos 78°}$

$c \approx 197.2$

$A = 12°$, $b = 192.9$, $c = 197.2$

12. $a^2 + b^2 = c^2$

$11^2 + 21^2 = c^2$

$\sqrt{562} = c$

$23.7 \approx c$

Find B.

$\tan B = \frac{b}{a}$

$\tan B = \frac{21}{11}$

$B = \tan^{-1} \frac{21}{11}$

$B \approx 62.4°$

Find A.

$A + 62.35402464 \approx 90$

$A \approx 27.64597536$

$c = 23.7$, $A = 27.6°$, $B = 62.4°$

13. $3.2° + B = 90°$

$B = 58°$

Find a.

$\sin A = \frac{a}{c}$

$\sin 32° = \frac{a}{13}$

$13 \sin 32° = a$

$6.9 \approx a$

Find b.

$\cos A = \frac{b}{c}$

$\cos 32° = \frac{b}{13}$

$13 \cos 32° = b$

$11.0 \approx b$

$B = 58°$, $a = 6.9$, $b = 11.0$

14a. $\tan x = \frac{1280}{2100}$

$x = \tan^{-1} \frac{1280}{2100}$

$x \approx 31.4°$

14b. $\tan 38° = \frac{1280}{x}$

$x \tan 38° = 1280$

$x = \frac{1280}{\tan 38°}$

$x \approx 1638.3$ ft

14c. $\tan 65° = \frac{1280}{x}$

$x \tan 65° = 1280$

$x = \frac{1280}{\tan 65°}$

$x \approx 596.9$ ft

Pages 309–312 Exercises

15. 90°

16. 120°, 300°

17. 30°, 330°

18. 90°, 270°

19. 225°, 315°

20. 135°, 315°

21. Sample answers: 30°, 150°, 390°, 510°

22. Let $A = \arccos \frac{4}{5}$. Then $\cos A = \frac{4}{5}$.

$\cos\left(\arccos \frac{4}{5}\right) = \frac{4}{5}$

23. Let $A = \tan^{-1} \frac{2}{3}$. Then $\tan A = \frac{2}{3}$.

$\tan\left(\tan^{-1} \frac{2}{3}\right) = \frac{2}{3}$

24. Let $A \cos^{-1} \frac{2}{5}$. Then $\cos A = \frac{2}{5}$.

$\sec A = \frac{1}{\cos A}$

$\sec A = \frac{1}{\frac{2}{5}}$

$\sec A = \frac{5}{2}$

$\sec\left(\cos^{-1} \frac{2}{5}\right) = \frac{5}{2}$

25. Let $A = \arcsin 1$. Then $\sin A = 1$.

$\csc A = \frac{1}{\sin A}$

$\csc A = \frac{1}{1}$ or 1

$\csc (\arcsin 1) = 1$

26. Let $A = \cos^{-1} \frac{5}{13}$. Then $\cos A = \frac{5}{13}$

$r^2 = x^2 + y^2$

$13^2 = 5^2 + y^2$

$144 = y^2$

$12 = y$

$\tan A = \frac{12}{5}$; $\tan\left(\cos^{-1} \frac{5}{13}\right) = \frac{12}{5}$

27. Let $A = \sin^{-1} \frac{2}{5}$. Then $\sin A = \frac{2}{5}$

$r^2 = x^2 + y^2$

$5^2 = x^2 + 2^2$

$21 = x^2$

$\sqrt{21} = x$

$\cos A = \frac{\sqrt{21}}{5}$; $\cos\left(\sin^{-1} \frac{2}{5}\right) = \frac{\sqrt{21}}{5}$

28. $\tan N = \frac{n}{m}$

$\tan N = \frac{15}{9}$

$N = \tan^{-1} \frac{15}{9}$

$N \approx 59.0°$

29. $\sin M = \frac{m}{p}$

$\sin M = \frac{8}{14}$

$M = \sin^{-1} \frac{8}{14}$

$M \approx 34.8°$

30. $\cos M = \frac{n}{p}$

$\cos M = \frac{22}{30}$

$M = \cos^{-1} \frac{22}{30}$

$M \approx 42.8°$

31. $\tan N = \frac{n}{m}$

$\tan N = \frac{18.8}{14.3}$

$N = \tan^{-1} \frac{18.8}{14.3}$

$N \approx 52.7°$

32. $\cos N = \frac{m}{p}$

$\cos N = \frac{7.2}{17.1}$

$N = \cos^{-1} \frac{7.2}{17.1}$

$N \approx 65.1°$

33. $\sin M = \frac{m}{p}$

$\sin M = \frac{32.5}{54.7}$

$M = \sin^{-1} \frac{32.5}{54.7}$

$M \approx 36.5°$

34. $\tan A = \frac{18}{24}$

$A = \tan^{-1} \frac{18}{24}$

$A \approx 36.9°$

$\tan B = \frac{24}{18}$

$B = \tan^{-1} \frac{24}{18}$

$B \approx 53.1°$

35. $\frac{1}{2}(14) = 7$

base angles:

$\tan A = \frac{8}{7}$

$A = \tan^{-1} \frac{8}{7}$

$A \approx 48.8°$

vertex angle $= 2m\angle B$

$\tan B = \frac{7}{8}$

$B = \tan^{-1} \frac{7}{8}$

$B \approx 41.2°$

$2m\angle B \approx 82.4°$

about 48.8°, 48.8°, and 82.4°

36. $a^2 + b^2 = c^2$

$21^2 + b^2 = 30^2$

$b = \sqrt{459}$

$b \approx 21.4$

$44.427004 + B \approx 90$

$B \approx 45.6°$

$\sin A = \frac{a}{c}$

$\sin A = \frac{21}{30}$

$A = \sin^{-1} \frac{21}{30}$

$A \approx 44.4°$

37. $35° + B = 90°$

$B = 55°$

$\tan A = \frac{a}{b}$

$\tan 35° = \frac{a}{8}$

$8 \tan 35° = a$

$5.6 \approx a$

$\cos A = \frac{b}{c}$

$\cos 35° = \frac{8}{c}$

$c \cos 35° = 8$

$c = \frac{8}{\cos 35°}$

$c \approx 9.8$

38. $A + 47° = 90°$

$A = 43°$

$\tan B = \frac{b}{a}$

$\tan 47° = \frac{12.5}{a}$

$a \tan 47° = 12.5$

$a = \frac{12.5}{\tan 47°}$

$a \approx 11.7$

$\sin B = \frac{b}{c}$

$\sin 47° = \frac{12.5}{c}$

$c \sin 47° = 12.5$

$c = \frac{12.5}{\text{sim } 47°}$

$c \approx 17.1$

39. $a^2 + b^2 = c^2$

$3.8^2 + 4.2^2 = c^2$

$\sqrt{32.08} = c$

$5.7 \approx c$

$42.13759477 + B \approx 90$

$B \approx 47.9°$

$\tan A = \frac{a}{b}$

$\tan A = \frac{3.8}{4.2}$

$A = \tan^{-1} \frac{3.8}{4.2}$

$A \approx 42.1°$

40. $a^2 + b^2 = c^2$

$a^2 + 3.7^2 = 9.5^2$

$a = \sqrt{76.56}$

$a \approx 8.7$

$A + 22.92175446 \approx 90$

$A \approx 67.1°$

$\sin B = \frac{b}{c}$

$\sin B = \frac{3.7}{9.5}$

$B = \sin^{-1} \frac{3.7}{9.5}$

$B \approx 22.9°$

41. $51.5° + B = 90°$

$B = 38.5°$

$\tan A = \frac{a}{b}$

$\tan 51.5° = \frac{13.3}{b}$

$b \tan 51.5° = 13.3$

$b = \frac{13.3}{\tan 51.5°}$

$b \approx 10.6$

$\sin A = \frac{a}{c}$

$\sin 51.5° = \frac{13.3}{c}$

$c \sin 51.5° = 13.3$

$c = \frac{13.3}{\sin 51.5°}$

$c \approx 17.0$

42. $A + 33° = 90°$

$A = 57°$

$\sin B = \frac{b}{c}$

$\sin 33° = \frac{b}{15.2}$

$15.2 \sin 33° = b$

$8.3 \approx b$

$\cos B = \frac{a}{c}$

$\cos 33° = \frac{a}{15.2}$

$15.2 \cos 33° = a$

$12.7 \approx a$

43. $14° + B = 90°$

$B = 76°$

$\sin A = \frac{a}{c}$

$\sin 14° = \frac{a}{9.8}$

$9.8 \sin 14° = a$

$2.4 \approx a$

$\cos A = \frac{b}{c}$

$\cos 14° = \frac{b}{9.8}$

$9.8 \cos 14° = b$

$9.5 \approx b$

44a. $\sin \theta = \frac{647}{1020}$

$\theta = \sin^{-1} \frac{647}{1020}$

$\theta \approx 39.4°$

44b. $\tan 39.4° \approx \frac{647}{x}$

$x \tan 39.4° \approx 647$

$x \approx \frac{647}{\tan 39.4°}$

$x \approx 788.5$ ft

45a. Since the sine function is the side opposite divided by the hypotenuse, the sine cannot be greater than 1.

45b. Since the secant function is the hypotenuse divided by the side opposite, the secant cannot be between 1 and −1.

45c. Since cosine function is the side adjacent divided by the hypotenuse, the cosine cannot be less than −1.

46. $10 - 6 = 4$

$\tan \theta = \frac{4}{15}$

$\theta = \tan^{-1} \frac{4}{15}$

$\theta \approx 14.9°$

47a. $\tan \theta = \frac{8}{100}$

$\theta = \tan^{-1} \frac{8}{100}$

$\theta \approx 4.6°$

47b. $\tan \theta = \frac{5}{100}$

$\theta = \tan^{-1} \frac{5}{100}$

$\theta \approx 2.9°$

48. $\tan \theta = \frac{45}{2200}$

$\theta = \tan^{-1} \frac{45}{2200}$

$\theta \approx 1.2°$

49. $\frac{65 \text{ miles}}{\text{hour}} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx 95.3 \frac{\text{feet}}{\text{second}}$

$\tan \theta = \frac{v^2}{gr}$

$\tan \theta = \frac{95.3^2}{32\ (1200)}$

$\theta = \tan^{-1} \frac{95.3^2}{32(1200)}$

$\theta \approx 13.3°$

50. $\frac{\sin \theta_i}{\sin \theta_r} = n$

$\frac{\sin 60°}{\sin \theta_r} = 2.42$

$\frac{\sin 60°}{2.42} = \sin \theta_r$

$0.3579 \approx \sin \theta_r$

$\sin^{-1} 0.3579 \approx \theta_r$

$21.0° \approx \theta_r$

51. Draw the altitude from Y to XZ. Call the point of intersection W.

$m\angle X + m\angle XYW = 90°$

$30° + m\angle XYW = 90°$

$m\angle XYW = 60°$

In $\triangle XYW$:

$\cos 30° = \frac{XW}{16}$ $\quad$ $\sin 30° = \frac{WY}{16}$

$16 \cos 30° = XW$ $\quad$ $16 \sin 30° = WY$

$13.9 \approx XW$ $\quad$ $8 = \mathrm{WY}$

In $\triangle ZYW$:

$\sin Z = \frac{8}{24}$ $\quad$ $\tan 19.5° = \frac{8}{WZ}$

$Z = \sin^{-1} \frac{8}{24}$ $\quad$ $WZ \tan 19.5° = 8$

$Z \approx 19.5°$ $\quad$ $WZ = \frac{8}{\tan 19.5°}$

$WZ \approx 22.6$

$\cos m\angle WYZ = \frac{8}{24}$

$m\angle WYZ = \cos^{-1} \frac{8}{24}$

$m\angle WYZ \approx 70.5°$

$Y = m\angle XYW + m\angle WYZ$ $\quad$ $y = XW + WZ$

$Y \approx 60° + 70.5°$ $\quad$ $y \approx 13.9 + 22.6$

$\mathrm{Y} \approx 130.5°$ $\quad$ $y \approx 36.5$

52. baseball stadium: $\quad$ football stadium:

$\tan 63° = \frac{1000}{x}$ $\quad$ $\tan 18° = \frac{1000}{y}$

$x \tan 63° = 1000$ $\quad$ $y \tan 18° = 1000$

$x = \frac{1000}{\tan 63°}$ $\quad$ $y = \frac{1000}{\tan 18°}$

$x \approx 509.5$ $\quad$ $y \approx 3077.7$

distance $= x + y$

distance $\approx 509.5 + 3077.7$

distance ≈ 3587.2 ft

53. $(FD)^2 + (DE)^2 = (FE)^2$

$7^2 + (DE)^2 = 15^2$

$(DE)^2 = 176$

$DE = \sqrt{176}$ or $4\sqrt{11}$

$\sin F = \frac{\text{side opposite}}{\text{hypotenuse}}$ $\quad$ $\cos F = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\sin F = \frac{4\sqrt{11}}{15}$ $\quad$ $\cos F = \frac{7}{15}$

$\tan F = \frac{\text{side opposite}}{\text{side adjacent}}$ $\quad$ $\csc F = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan F = \frac{4\sqrt{11}}{7}$ $\quad$ $\csc F = \frac{15}{4\sqrt{11}}$ or $\frac{15\sqrt{11}}{44}$

$\sec F = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\quad$ $\cot F = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec F = \frac{15}{7}$ $\quad$ $\cot F = \frac{7}{4\sqrt{11}}$ or $\frac{7\sqrt{11}}{44}$

54. Use TABLE feature of a graphing calculator.

$-0.3, 1.4, 4.3$

55. x-axis
$y^3 - x^2 = 2$
$(-y)^3 - x^2 = 2$
$-y^3 - x^2 = 2$; no

y-axis
$y^3 - x^2 = 2$
$y^3 - (-x)^2 = 2$
$y^3 - x^2 = 2$; yes

$y = x$
$y^3 - x^2 = 2$
$(x)^3 - (y)^2 = 2$
$x^3 - y^2 = 2$; no

$y = -x$
$y^3 - x^2 = 2$
$(-x)^3 - (-y)^2 = 2$
$-x^3 - y^2 = 2$; no

y-axis

56. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & -5 & -3 & -1 & -2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1(-5) + 0(-3) & -1(-5) + 0(4) & -1(-3) + 0(6) & -1(-1) + 0(3) & -1(-2) + 0(-2) \\ 0(-5) + 1(-3) & 0(-5) + 1(4) & 0(-3) + 1(6) & 0(-1) + 1(3) & 0(-2) + 1(-2) \end{bmatrix}$

$= \begin{bmatrix} 5 & 5 & 3 & 1 & 2 \\ -3 & 4 & 6 & 3 & -2 \end{bmatrix}$

$(5, -3), (5, 4), (3, 6), (1, 3), (2, -2)$

57. $\begin{bmatrix} 4 & -3 & 2 \\ 8 & -2 & 0 \\ 9 & 6 & -3 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -2 \\ -5 & 1 & 1 \\ -7 & 2 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4 + (-2) & -3 + 2 & 2 + (-2) \\ 8 + (-5) & -2 + 1 & 0 + 1 \\ 9 + (-7) & 6 + 2 & -3 + (-2) \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 0 \\ 3 & -1 & 1 \\ 2 & 8 & -5 \end{bmatrix}$

58. $m = \frac{22.2 - 42.5}{1950 - 1880}$

$m = \frac{-20.3}{70}$ or -0.29

$y - 22.2 = -0.29(x - 1950)$

$y = -0.29x + 587.7$

59. $2x + 5y - 10 = 0$

$5y = -2x + 10$

$y = -\frac{2}{5}x + 2$

$-\frac{2}{5}; 2$

60. $\frac{\mathrm{a}}{\mathrm{a} + c} \cdot b + \frac{c}{a + c} \cdot d + 10 = \frac{ab + cd}{a + c} + 10$

The correct choice is A.

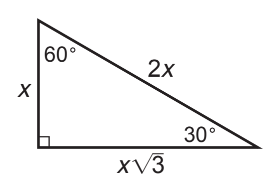
5-6 The Law of Sines

Page 316 Check for Understanding

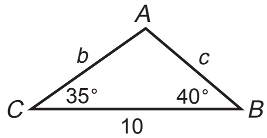
1. $\frac{x}{\sin 30°} \stackrel{?}{=} \frac{x\sqrt{3}}{\sin 60°} \stackrel{?}{=} \frac{2x}{\sin 90°}$

$\frac{x}{\frac{1}{2}} \stackrel{?}{=} \frac{x\sqrt{3}}{\frac{\sqrt{3}}{2}} \stackrel{?}{=} \frac{2x}{1}$

$2x = 2x = 2x$



2. Sample answer:



3. Area of $WXYZ$ = Area of triangle ZWY + Area of triangle XYW.
$m\angle X = m\angle Z$

triangle ZWY: triangle XYW:

$K = \frac{1}{2ab}\sin Z$ $\quad k = \frac{1}{2}ab\sin X$

$K = \frac{1}{2ab}\sin X$

$K = \frac{1}{2}ab\sin X + \frac{1}{2}ab\sin X$

$K = ab\sin X$

4. Both; if the measures of two angles and a non-included side are known or if the measures of two angles and the included side are known, the triangle is unqiue.

5. $C = 180° - (40° + 59°)$ or $81°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\qquad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 40°} = \frac{14}{\sin 81°}$ $\qquad \frac{b}{\sin 59°} = \frac{14}{\sin 81°}$

$a = \frac{14\sin 40°}{\sin 81°}$ $\qquad b = \frac{14\sin 59°}{\sin 81°}$

$a \approx 9.111200533$ $\qquad b \approx 12.14992798$

$C = 81°, a = 9.1, b = 12.1$

6. $C = 180° - (27.3° + 55.9°)$ or $96.8°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\qquad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 55.9°} = \frac{8.6}{\sin 27.3°}$ $\qquad \frac{c}{\sin 96.8°} = \frac{8.6}{\sin 27.3°}$

$b = \frac{8.6\sin 55.9°}{\sin 27.3°}$ $\qquad c = \frac{8.6\sin 96.8°}{\sin 27.3°}$

$b \approx 15.52671055$ $\qquad c \approx 18.61879792$

$C = 96.8°, b = 15.5, c = 18.6$

7. $A = 180° - (17°\,55' + 98°\,15')$ or $63°\,50'$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 98°\,15'} = \frac{17}{\sin 63°\,50'}$

$c = \frac{17\sin 98°\,15'}{\sin 63°\,50'}$

$c \approx 18.7$

8. $K = \frac{1}{2}\,bc\sin A$

$K = \frac{1}{2}(14)(12)\sin 78°$

$K \approx 82.2$ units^2

9. $C = 180° - (22° + 105°)$ or $53°$

$K = \frac{1}{2}\,b^2\,\frac{\sin A\sin C}{\sin B}$

$K = \frac{1}{2}(14)^2\,\frac{\sin 22°\sin 53°}{\sin 105°}$

$K \approx 30.4$ units^2

10. Let d = the distance from the fan to the pitcher's mound.
$\theta = 180° - (24°\,12' + 5°\,42')$ or $150°\,6'$

$\frac{d}{\sin 150°\,6'} = \frac{60.5}{\sin 5°\,42'}$

$d = \frac{60.5\sin 150°\,6'}{\sin 5°\,42'}$

$d \approx 303.7$ ft

Pages 316–318 Exercises

11. $B = 180° - (40° + 70°)$ or $70°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\qquad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 70°} = \frac{20}{\sin 40°}$ $\qquad \frac{c}{\sin 70°} = \frac{20}{\sin 40°}$

$b = \frac{20\sin 70°}{\sin 40°}$ $\qquad c = \frac{20\sin 70°}{\sin 40°}$

$b \approx 29.238044$ $\qquad c \approx 29.238044$

$B = 70°, b = 29.2, c = 29.2$

12. $A = 180° - (100° + 50°)$ or $30°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\qquad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 30°} = \frac{30}{\sin 50°}$ $\qquad \frac{b}{\sin 100°} = \frac{30}{\sin 50'}$

$a = \frac{30\sin 30°}{\sin 50°}$ $\qquad b = \frac{30\sin 100°}{\sin 50°}$

$a \approx 19.58110934$ $\qquad b \approx 38.56725658$

$A = 30°, a = 19.6, b = 38.6$

13. $C = 180° - (25° + 35°)$ or $120°$

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\qquad \frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{a}{\sin 25°} = \frac{12}{\sin 35°}$ $\qquad \frac{c}{\sin 120°} = \frac{12}{\sin 35°}$

$a = \frac{12\sin 25°}{\sin 35°}$ $\qquad c = \frac{12\sin 120°}{\sin 35°}$

$a \approx 8.84174945$ $\qquad c \approx 18.11843058$

$C = 120°, a = 8.8, c = 18.1$

14. $C = 180° - (65° + 50°)$ or $65°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\qquad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 65°} = \frac{12}{\sin 65°}$ $\qquad \frac{b}{\sin 50°} = \frac{12}{\sin 65°}$

$a = \frac{12\sin 65°}{\sin 65°}$ $\qquad b = \frac{12\sin 50°}{\sin 65°}$

$a = 12$ $\qquad b \approx 10.14283828$

$C = 65°, a = 12, b = 10.1$

15. $A = 180° - (24.8° + 61.3°)$ or $93.9°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\qquad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{b}{\sin 24.8°} = \frac{8.2}{\sin 93.9°}$ $\qquad \frac{c}{\sin 61.3°} = \frac{8.2}{\sin 93.9°}$

$b = \frac{8.2\sin 24.8°}{\sin 93.9°}$ $\qquad c = \frac{8.2\sin 61.3°}{\sin 93.9°}$

$b \approx 3.447490503$ $\qquad c \approx 7.209293255$

$A = 93.9°, b = 3.4, c = 7.2$

16. $B = 180° - (39°\,15' + 64°\,45')$ or $76°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\qquad \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 39'\,51'} = \frac{19.3}{\sin 64°\,45'}$ $\qquad \frac{b}{\sin 76°} = \frac{19.3}{\sin 64°\,45'}$

$a = \frac{19.3\sin 39°\,15'}{\sin 64°\,45'}$ $\qquad b = \frac{19.3\sin 76°}{\sin 64°\,45'}$

$a \approx 13.50118124$ $\qquad b \approx 20.7049599$

$B = 76°, a = 13.5, b = 20.7$

17. $C = 180° - (37°\,20' + 51°\,30')$ or $91°\,10'$

$\frac{a}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 51°\,30'} = \frac{125}{\sin 91°\,10'}$

$b = \frac{125\sin 51°\,31'}{\sin 91°\,10'}$

$b \approx 97.8$

18. A $= 180° - (29°\,34' + 23°\,48')$ or $126°\,38'$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 126°\,38'} = \frac{11}{\sin 29°\,34'}$

$a = \frac{11\sin 126°\,38'}{\sin 29°\,34'}$

$a \approx 17.9$

19. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(14)(9) \sin 28°$

$K \approx 29.6$ units^2

20. $A = 180° - (37° + 84°)$ or $59°$

$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(5)^2 \frac{\sin 37° \sin 84°}{\sin 59°}$

$K \approx 8.7$ units^2

21. $C = 180° - (15° + 113°)$ or $52°$

$K = \frac{1}{2} b^2 \frac{\sin A \sin C}{\sin B}$

$K = \frac{1}{2}(7)^2 \frac{\sin 15° \sin 52°}{\sin 113°}$

$K \approx 5.4$ units^2

22. $K = \frac{1}{2}bc \sin A$

$K = \frac{1}{2}(146.2)(209.3) \sin 62.2°$

$K \approx 13{,}533.9$ units^2

23. $K = \frac{1}{2}ac \sin B$

$K = \frac{1}{2}(12.7)(5.8) \sin 42.8°$

$K \approx 25.0$ units^2

24. $B = 180° - (53.8° + 65.4°)$ or $60.8°$

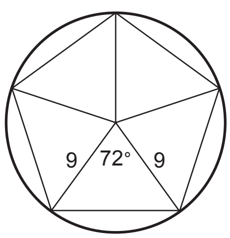
$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$

$K = \frac{1}{2}(19.2)^2 \frac{\sin 60.8° \sin 65.4°}{\sin 53.8°}$

$K \approx 181.3$ units^2

25. $K = ab \sin X$ (formula from Exercise 3)
$K = (14)(20) \sin 57°$
$K \approx 234.8$ cm^2

26.

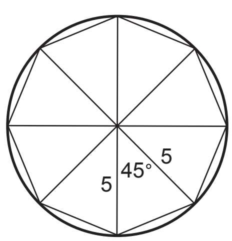


Area of pentagon = 5 · Area of triangle
$360° \div 5 = 72°$

$K = \frac{1}{2}(9)(9) \sin 72°$ $\quad$ $5K \approx 5(38.51778891)$

$K \approx 38.51778891$ $\quad$ $5K \approx 192.6$ in^2

27.



Area of octagon = 8 · Area of triangle
$360° \div 8 = 45°$

$K = \frac{1}{2}(5)(5) \sin 45°$ $\quad$ $8K \approx 8(8.838834765)$

$K \approx 8.838834765$ $\quad$ $8K \approx 70.7$ ft^2

28a. $180° - (95° + 40°) = 45°$

28b. $\frac{x}{\sin 95°} = \frac{80}{\sin 45°}$ $\quad$ $\frac{y}{\sin 40°} = \frac{80}{\sin 45°}$

$x = \frac{80 \sin 95°}{\sin 45°}$ $\quad$ $y = \frac{80 \sin 40°}{\sin 45°}$

$x \approx 112.7065642$ $\quad$ $y \approx 72.72311643$

about 112.7 ft and 72.7 ft

28c. $P \approx 112.7 + 72.7 + 80$
$P \approx 265.4$ ft

29. Applying the Law of Sines, $\frac{m}{\sin M} = \frac{n}{\sin N}$ and $\frac{r}{\sin R} = \frac{s}{\sin S}$. Thus $\sin M = \frac{n \sin N}{n}$ and $\sin R = \frac{r \sin S}{s}$. Since $\angle M \cong \angle R$, $\sin M = \sin R$ and $\frac{m \sin N}{n} = \frac{r \sin S}{s}$. However, $\angle N \cong \angle S$ and $\sin N = \sin S$, so $\frac{m}{n} = \frac{r}{s}$ and $\frac{m}{r} = \frac{n}{s}$. Similar proportions can be derived for p and t. Therefore, $\triangle MNP \cong \triangle RST$.

30. $360° \div 5 = 72°$

triangle: $K = \frac{1}{2}(300)(300)\sin 72°$

$K \approx 42{,}797.54323$

pentagon: $5K \approx 5(42{,}797.54323)$

$5K \approx 213{,}987.7$ ft

31a. $\theta = 180° - (20°\ 15' + 62°\ 30')$ or $97°\ 15'$
Let x = the distance from the balloon to the soccer fields.

$\frac{x}{\sin 62°\ 30'} = \frac{4}{\sin 97°\ 15'}$

$x = \frac{4 \sin 62°\ 30'}{\sin 97°\ 15'}$

$x \approx 3.6$ mi

31b. $\theta = 180° - (20°\ 15' + 62°\ 30')$ or $97°\ 15'$
Let y = the distance from the balloon to the football field.

$\frac{4}{\sin 20°\ 15'} = \frac{4}{\sin 97°\ 15'}$

$y = \frac{4 \sin 20°\ 15'}{\sin 97°\ 15'}$

$y \approx 1.4$ mi

32. $180° - 30° = 150°$
$\theta = 180° - (26.8° + 150°)$ or $3.2°$
Let x = the length of the track.

$\frac{x}{\sin 26.8°} = \frac{100}{\sin 3.2°}$

$x = \frac{100 \sin 26.8°}{\sin 3.2°}$

$x \approx 807.7$ ft

33a. Let x = the distance of the second part of the flight.
$\theta = 180° - (13° + 160°)$ or $7°$

$\frac{x}{\sin 13°} = \frac{80}{\sin 7°}$

$x = \frac{80 \sin 13°}{\sin 7°}$

$x \approx 147.6670329$

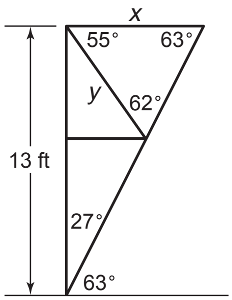
distance of flight $\approx 80 + 147.7$ or about 227.7 mi

33b. Let y = the distance of a direct flight.

$\frac{y}{\sin 160°} = \frac{80}{\sin 7°}$

$y = \frac{80 \sin 160°}{\sin 7°}$

$y \approx 224.5$ mi

34. 

$90° - 63° = 27°$
$180° - (55° + 63°) = 62°$
Let x = the vertical distance.
Let y = the length of the overhang.

$\frac{x}{\sin 27°} = \frac{13}{\sin 63°}$ $\quad$ $\frac{y}{\sin 63°} = \frac{66}{\sin 62°}$

$x = \frac{13 \sin 27°}{\sin 63°}$ $\quad$ $y = \frac{6.6 \sin 63°}{\sin 62°}$

$x \approx 6.623830843$ $\quad$ $y \approx 6.684288563$

about 6.7 ft

35a. $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{b} = \frac{\sin A}{\sin B}$

35b. $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{c} = \frac{\sin A}{\sin C}$

$\frac{a}{c} - 1 = \frac{\sin A}{\sin C} - 1$

$\frac{a}{c} - \frac{c}{c} = \frac{\sin A}{\sin C} - \frac{\sin C}{\sin C}$

$\frac{a - c}{c} = \frac{\sin A - \sin C}{\sin C}$

35c. From Exercise 34b, $\frac{a - c}{c} = \frac{\sin A - \sin C}{\sin C}$ or $\frac{\sin A - \sin C}{a - c} = \frac{\sin C}{c}$.

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{c} = \frac{\sin A}{\sin C}$

$\frac{a}{c} + 1 = \frac{\sin A}{\sin C} + 1$

$\frac{a}{c} + \frac{c}{c} = \frac{\sin A}{\sin C} + \frac{\sin C}{\sin C}$

$\frac{a + c}{c} = \frac{\sin A + \sin C}{\sin C}$

$\frac{\sin C}{c} = \frac{\sin A + \sin C}{a + c}$

Therefore, $\frac{\sin A - \sin C}{a - c} = \frac{\sin A + \sin C}{a + c}$

or $\frac{a + c}{a - c} = \frac{\sin A + \sin C}{\sin A - \sin C}$.

35d. $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{b} = \frac{\sin A}{\sin B}$

$\frac{a}{b} + 1 = \frac{\sin A}{\sin B} + 1$

$\frac{a}{b} + \frac{b}{b} = \frac{\sin A}{\sin B} + \frac{\sin B}{\sin B}$

$\frac{a + b}{b} = \frac{\sin A + \sin B}{\sin B}$

$\frac{b}{a + b} = \frac{\sin B}{\sin A + \sin B}$

36. $\tan \theta = \frac{45}{20}$

$\theta = \tan^{-1} \frac{45}{20}$

$\theta \approx 66.0°$

37. $\sin \theta = \frac{y}{r}$ $\quad$ $r^2 = x^2 + y^2$

$\sin \theta = -\frac{1}{6}$ $\quad$ $6^2 = x^2 + (-1)^2$

$y = -1, r = 6$ $\quad$ $35 = x^2$

$\pm\sqrt{35} = x$

Quadrant IV, so $x = \sqrt{35}$

$\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$ $\quad$ $\csc \theta = \frac{r}{y}$

$\cos \theta = \frac{\sqrt{35}}{6}$ $\quad$ $\tan \theta = \frac{-1}{\sqrt{35}}$ $\quad$ $\csc \theta = \frac{6}{-1}$ or -6

$\tan \theta = -\frac{\sqrt{35}}{35}$

$\sec \theta = \frac{r}{x}$ $\quad$ $\cot \theta = \frac{x}{y}$

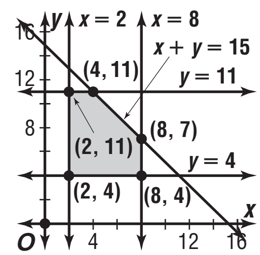
$\sec \theta = \frac{6}{\sqrt{35}}$ $\quad$ $\cot \theta = \frac{\sqrt{35}}{-1}$ or $-\sqrt{35}$

$\sec \theta = \frac{6\sqrt{35}}{35}$

38. $83° + 360k°$

39. Let x = standard carts and let y = deluxe carts.

$2 \le x \le 8$
$4 \le y \le 11$
$x + y \le 15$



$M(x, y) = 100x + 250y$
$M(2, 4) = 100(2) + 250(4)$ or 1200
$M(2, 11) = 100(2) + 250(11)$ or 2950
$M(4, 11) = 100(4) + 250(11)$ or 3150
$M(8, 7) = 100(8) + 250(7)$ or 2550
$M(8, 4) = 100(8) + 250(4)$ or 1800
4 standard carts, 11 deluxe carts

40.
$$\begin{array}{r} 4x + y + 2z = 0 \\ \underline{3x + 4y - 2z = 20} \\ 7x + 5y \quad\quad = 20 \end{array}$$

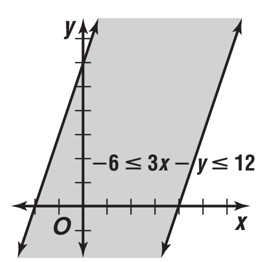
$3(3x + 4y - 2z) = 3(20)$
$2(-2x + 5y + 3z) = 2(14)$
$\downarrow$

$$\begin{array}{r} 9x + 12y - 6z = 60 \\ \underline{-4x + 10y + 6z = 28} \\ 5x + 22y \quad\quad = 88 \end{array}$$

$-5(7x + 5y) = -5(20)$ $\rightarrow$ $-35x - 25y = -100$
$7(5x + 22y) = 7(88)$ $\quad$ $\underline{35x + 154y = 616}$
$129y = 516$
$y = 4$

$7x + 5y = 20$ $\quad$ $4x + y + 2z = 0$
$7x + 5(4) = 20$ $\quad$ $4(0) + 4 + 2z = 0$
$x = 0$ $\quad$ $z = -2$
$(0, 4, -2)$

41. $-6 \le 3x - y$ $\quad$ $3x - y \le 12$
$y \le 3x + 6$ $\quad$ $y \ge 3x - 12$



42. Area of one face of the small cube = 1^2 or 1 in^2.
Surface area of the small cube = $6 \cdot 1$ or 6 in^2.
Area of one face of large cube = 2^2 or 4 in^2.
Surface area of large cube = $6 \cdot 4$ or 24 in^2.
Surface area of all small cubes = $8 \cdot 6$ or 48 in^2.
The difference in surface areas is 48 in^2 − 24 in^2 or 24 in^2.
The correct choice is A.

Page 319 History of Mathematics

1. See students' work; the sum is greater than 180°. In spherical geometry, the sum of the angles of a triangle can exceed 180°.
2. See students' work. Sample answer: Postulate 4 states that all right angles are equal to one another.
3. See students' work.

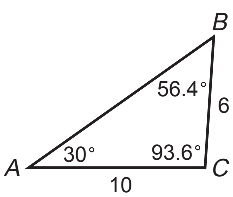
5-7 The Ambiguous Case for the Law of Sines

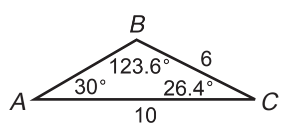
Page 323 Graphing Calculator Exploration

1. $B = 44.1°$, $C = 23.9°$, $c = 1.8$
2. $B = 52.7°$, $C = 76.3°$, $b = 41.0$; $B = 25.3°$, $C = 103.7°$, $b = 22.0$
3. The answers are slightly different.
4. Answers will vary if rounded numbers are used to find some values.

Page 324 Check for Understanding

1. A triangle cannot exist if $m\angle A < 90°$ and $a < b \sin A$ or if $m\angle A \geq 90°$ and $a \leq b$.
2.





$$\frac{6}{\sin 30°} = \frac{10}{\sin B}$$
$$\sin B = \frac{10 \sin 30°}{6}$$
$$B = \sin^{-1}\left(\frac{10 \sin 30°}{6}\right)$$
$$B \approx 56.44269024$$
$$C \approx 180° - (30° + 56.4°)$$
$$\approx 93.6°$$

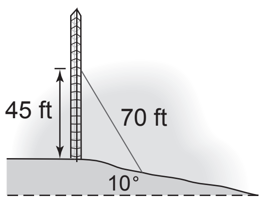
$$180° - \propto\ \approx 180° - 56.4°$$
$$\approx 123.6°$$
$$C \approx 180° - (30° + 123.6°)$$
$$\approx 26.4°$$

3. Step 1: Determine that there is one solution for the triangle.
   Step 2: Use the Law of Sines to solve for B.
   Step 3: Subtract the sum of 120 and B from 180 to find C.
   Step 4: Use the Law of Sines to solve for c.
4. Since $113° \geq 90°$, consider Case II.
   $15 \geq 8$; 1 solution
5. Since $44° < 90°$, consider Case I.
   $a \sin B = 23 \sin 44°$
   $a \sin B \approx 23\ (0.6947)$
   $a \sin B \approx 15.97714252$
   $12 < 16.0$; 0 solutions
6. Since $17° < 90°$, consider Case I.
   $a \sin C = 10 \sin 17°$
   ≈ 2.923717047
   $2.9 < 10 < 11$; 1 solution
   $$\frac{c}{\sin C} = \frac{a}{\sin A}$$
   $$\frac{11}{\sin 17°} = \frac{10}{\sin A}$$
   $$\sin A = \frac{10 \sin 17°}{11}$$
   $$A = \sin^{-1}\left(\frac{10 \sin 17°}{11}\right)$$
   $$A \approx 15.41404614$$
   $B \approx 180° - (15.4° + 17°)$ or about 147.6°
   $$\frac{c}{\sin C} = \frac{b}{\sin B}$$
   $$\frac{11}{\sin 17°} \approx \frac{b}{\sin 147.6°}$$
   $$b \approx \frac{11 \sin 147.6°}{\sin 17°}$$
   $$b \approx 20.16738057$$
   $A = 15.4°$, $B = 147.6°$, $b = 20.2$
7. Since $140° \geq 90°$, consider Case II.
   $3 \leq 10$; no solutions
8. Since $38° < 90°$, consider Case I.
   $b \sin A = 10 \sin 38°$
   $b \sin A \approx 6.156614753$
   $6.2 < 8 < 10$; 2 solutions
   $$\frac{a}{\sin A} = \frac{b}{\sin B}$$
   $$\frac{8}{\sin 38°} = \frac{10}{\sin B}$$
   $$\sin B = \frac{10 \sin 38°}{8}$$
   $$B = \sin^{-1}\left(\frac{10 \sin 38°}{8}\right)$$
   $$B \approx 50.31590502$$
   $180° - \propto\ \approx 180° - 50.3°$ or 129.7°
   Solution 1
   $C \approx 180° - (50.3° + 38°)$ or 91.7°
   $$\frac{a}{\sin A} = \frac{c}{\sin C}$$
   $$\frac{8}{\sin 38°} \approx \frac{c}{\sin 91.7°}$$
   $$c \approx \frac{8 \sin 91.7°}{\sin 38°}$$
   $$c \approx 12.98843472$$
   Solution 2
   $C \approx 180° - (129.7° + 38°)$ or 12.3°
   $$\frac{a}{\sin A} = \frac{c}{\sin C}$$
   $$\frac{8}{\sin 38°} \approx \frac{c}{\sin 12.3°}$$
   $$c \approx \frac{8 \sin 12.3°}{\sin 38°}$$
   $$c \approx 2.768149638$$
   $B = 50.3°$, $C = 91.7°$, $c = 13.0$; $B = 129.7°$, $C = 12.3°$, $c = 2.8$

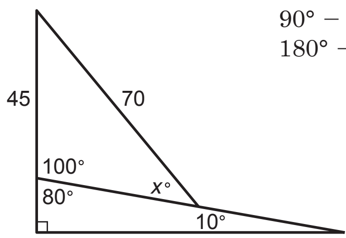
9. Since $130° \geq 90°$, consider Case II.
$17 > 5$; 1 solution

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{17}{\sin 130°} = \frac{5}{\sin B}$$
$$\sin B = \frac{5 \sin 130°}{17}$$
$$B = \sin^{-1}\left(\frac{5 \sin 130°}{17}\right)$$
$$B \approx 13.02094264$$
$$A \approx 180° - (13.0 + 130°) \text{ or } 37.0°$$
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{17}{\sin 130°} \approx \frac{a}{\sin 37.0°}$$
$$a \approx \frac{17 \sin 37.0°}{\sin 130°}$$
$$a \approx 13.35543321$$
$A = 37.0°$, $B = 13.0°$, $a = 13.4$

10a.



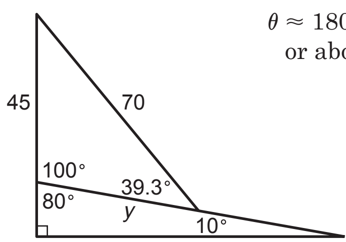
10b.



$90° - 10° = 80°$
$180° - 80° = 100°$

$$\frac{70}{\sin 100°} = \frac{45}{\sin x}$$
$$\sin x = \frac{45 \sin 100°}{70}$$
$$x = \sin^{-1}\left(\frac{45 \sin 100°}{70}\right)$$
$$x \approx 39.3°$$

10c.



$\theta \approx 180° - (100° - 39.3°)$ or about $40.7°$

$$\frac{y}{\sin 40.7°} = \frac{70}{\sin 100°}$$
$$y = \frac{70 \sin 40.7°}{\sin 100°}$$
$$y \approx 46.4 \text{ ft}$$

Pages 324–326 Exercises

11. Since $57° < 90°$, consider Case I.
$b \sin A = 19 \sin 57°$
$b \sin A \approx 15.93474079$
$11 < 15.9$; 0 solutions

12. Since $30° < 90°$, consider Case I.
$c \sin A = 26 \sin 30°$
$c \sin A = 13$
$13 = 13$; 1 solution.

13. Since $61° < 90°$, consider Case I.
$a \sin B = 12 \sin 61°$
$a \sin B \approx 10.49543649$
$8 < 10.5$; 0 solutions

14. two angles are given; 1 solution

15. Since $100° \geq 90°$, consider Case II.
$15 < 18$; 0 solutions

16. Since $37° < 90°$, consider Case I.
$a \sin B = 32 \sin 37°$
$a \sin B \approx 19.25808074$
$27 > 19.3$; 2 solutions

17. Since $65° < 90°$, consider Case I.
$b \sin A = 57 \sin 65°$
$b \sin A \approx 51.65954386$
$55 > 51.7$; 2 solutions

18. Since $150° \geq 90°$, consider Case II.
$6 \leq 8$; no solution

19. Since $58° < 90°$, consider Case I.
$b \sin A = 29 \sin 58°$
$b \sin A \approx 24.59339479$
$26 > 24.6$; 2 solutions

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{26}{\sin 58°} = \frac{29}{\sin B}$$
$$\sin B = \frac{29 \sin 58°}{26}$$
$$B = \sin^{-1}\left(\frac{29 \sin 58°}{26}\right)$$
$$B \approx 71.06720496$$
$$180° - \propto \approx 180° - 71.1° \text{ or } 108.9°$$

Solution 1
$$C \approx 180° - (58° + 71.1°) \text{ or } 50.9°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{26}{\sin 58°} \approx \frac{c}{\sin 50.9°}$$
$$c \approx \frac{26 \sin 50.9°}{\sin 58°}$$
$$c \approx 23.80359004$$

Solution 2
$$C \approx 180° - (58° + 108.9)° \text{ or } 13.1°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{26}{\sin 58°} \approx \frac{c}{\sin 13.1°}$$
$$c \approx \frac{26 \sin 13.1°}{\sin 58°}$$
$$c \approx 6.931727606$$
$B = 71.1°$, $C = 50.9°$, $c = 23.8$; $B = 108.9°$, $C = 13.1°$, $c = 6.9$

20. Since $30° < 90°$, consider Case I.
$b \sin A = 8 \sin 30°$
$b \sin A = 4$
$4 = 4$; 1 solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{4}{\sin 30°} = \frac{8}{\sin B}$$
$$\sin B = \frac{8 \sin 30°}{4}$$
$$B = \sin^{-1}\left(\frac{8 \sin 30°}{4}\right)$$
$$B = 90$$

$C = 180° - (30° - 90°)$ or $60°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{4}{\sin 30°} = \frac{c}{\sin 60°}$$
$$C = \frac{4 \sin 60°}{\sin 30°}$$
$$C \approx 6.92820323$$

$B = 90°$; $C = 60°$, $c = 6.9$

21. Since $70° < 90°$, consider Case I.
$a \sin C = 25 \sin 70°$
$a \sin C \approx 23.49231552$
$24 > 23.5$; 2 solutions

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{24}{\sin 70°} = \frac{25}{\sin A}$$
$$\sin A = \frac{25 \sin 70°}{24}$$
$$A = \sin^{-1}\left(\frac{25 \sin 70°}{24}\right)$$
$$A \approx 78.1941432$$

$180° - \propto \approx 180° - 79.2°$ or $101.8°$
Solution 1
$B \approx 180° - (70° + 79.2°)$ or $31.8°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{24}{\sin 70°} \approx \frac{b}{\sin 31.8°}$$
$$b \approx \frac{24 \sin 31.8°}{\sin 70°}$$
$$b \approx 13.46081025$$

Solution 2
$B \approx 180° - (70° + 101.8°)$ or $8.2°$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{24}{\sin 70°} \approx \frac{b}{\sin 8.2°}$$
$$b \approx \frac{24 \sin 8.2°}{\sin 70°}$$
$$b \approx 3.640196918$$

$A = 78.2°$, $B = 31.8°$, $b = 13.5$; $A = 101.8°$, $B = 8.2°$, $b = 3.6$

22. $C = 180° - (40° + 60°)$ or $80°$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{20}{\sin 80°} = \frac{a}{\sin 40°} \qquad \frac{20°}{\sin 80°} = \frac{b}{\sin 60°}$$
$$a = \frac{20 \sin 40°}{\sin 80°} \qquad b = \frac{20 \sin 60°}{\sin 80°}$$
$$a \approx 13.05407289 \qquad b \approx 17.58770483$$

$C = 80°$, $a = 13.1$, $b = 17.6$

23. Since $90° \geq 90°$; consider Case II.
$12 \leq 14$; no solution

24. Since $36° < 90°$, consider Case I.
$c \sin B = 30 \sin 36°$
$c \sin B \approx 17.63355757$
$19 > 17.6$; 2 solutions

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{19}{\sin 36°} = \frac{30}{\sin C}$$
$$\sin C = \frac{30 \sin 36°}{19}$$
$$C = \sin^{-1}\left(\frac{30 \sin 36°}{19}\right)$$
$$C \approx 68.1377773$$

$180° - \propto \approx 180° - 68.1°$ or $111.9°$
Solution 1
$A \approx 180° - (36° + 68.1°)$ or $75.9°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{19}{\sin 36°} \approx \frac{a}{\sin 75.9°}$$
$$a \approx \frac{19 \sin 75.9°}{\sin 36°}$$
$$a \approx 31.34565276$$

Solution 2
$A \approx 180° - (36° + 111.9°)$ or $32.1°$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{19}{\sin 36°} \approx \frac{a}{\sin 32.1°}$$
$$a \approx \frac{19 \sin 32.1°}{\sin 36}$$
$$a \approx 17.1953669$$

$A = 75.9°$, $C = 68.1°$, $a = 31.3$; $A = 32.1°$, $C = 111.9°$, $a = 17.2$

25. Since $107.2° \geq 90°$, consider Case II.
$17.2 > 12.2$; 1 solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{17.2}{\sin 107.2°} = \frac{12.2}{\sin C}$$
$$\sin C = \frac{12.2 \sin 107.2°}{17.2}$$
$$C = \sin^{-1}\left(\frac{12.2 \sin 107.2°}{17.2}\right)$$
$$C \approx 42.65491459$$

$B \approx 180° - (107.2° + 42.7°)$ or $30.1°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{17.2}{\sin 107.2°} \approx \frac{b}{\sin 30.1°}$$
$$b \approx \frac{17.2 \sin 30.1°}{\sin 107.2°}$$
$$b \approx 9.042067456$$

$B = 30.1°$, $C = 42.7°$, $b = 9.0$

26. Since $76° < 90°$, consider Case I.
$b \sin A = 20 \sin 76°$
$b \sin A \approx 19.40591453$
$5 < 19.4$; no solution

27. Since $47° < 90°$, consider Case I.

$16 \geq 10$; 1 solution

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{16}{\sin 47°} = \frac{10}{\sin A}$

$\sin A = \frac{10 \sin 47°}{16}$

$A = \sin^{-1}\left(\frac{10 \sin 47°}{16}\right)$

$A \approx 27.19987995$

$B \approx 180° - (47° - 27.2)$ or $105.8°$

$\frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{16}{\sin 47°} = \frac{b}{\sin 105.8°}$

$b = \frac{16 \sin 105.8°}{\sin 47°}$

$b \approx 21{,}0506609$

$A = 27.2°, B = 105.8°, b = 21.1$

28. Since $40° < 90°$, consider Case I.

$c \sin B = 60 \sin 40°$

$c \sin B \approx 38.56725658$

$42 > 38.6$; 2 solutions

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{42}{\sin 40°} = \frac{60}{\sin C}$

$\sin C = \frac{60 \sin 40°}{42}$

$C = \sin^{-1}\left(\frac{60 \sin 40°}{42}\right)$

$C \approx 66.67417652$

$180° - \propto \approx 180° - 66.7°$ or $113.3°$

Solution 1

$A \approx 180° - (40° + 66.7°)$ or $73.3°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{42}{\sin 40°} \approx \frac{a}{\sin 73.3°}$

$a \approx \frac{42 \sin 73.3°}{\sin 40°}$

$a \approx 62.58450564$

Solution 2

$A \approx 180° - (40° + 113.3°)$ or $26.7°$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{42}{\sin 40°} = \frac{a}{\sin 26.7°}$

$a = \frac{42 \sin 26.7°}{\sin 40°}$

$a \approx 29.33237132$

$A = 73.3°, C = 66.7°, a = 62.6; A = 26.7°,$
$C = 113.3°, a = 29.3$

29. Since $125.3° \geq 90°$, consider Case II.

$32 \leq 40$; no solution

30. $\frac{21.7}{\sin 57.4°} = \frac{19.3}{\sin x}$

$\sin x = \frac{19.3 \sin 57.4°}{21.7}$

$x = \sin^{-1}\left(\frac{19.3 \sin 57.4°}{21.7}\right)$

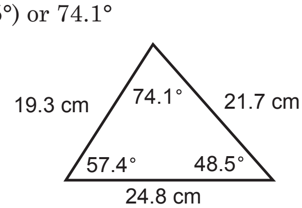
$x \approx 48.52786934$

$\theta \approx 180 - (57.4° + 48.5°)$ or $74.1°$

$\frac{21.7}{\sin 57.4°} \approx \frac{y}{\sin 74.1°}$

$y \approx \frac{21.7 \sin 74.1°}{\sin 57.4°}$

$y \approx 24.76922417$



31. $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{15}{\sin 29°} = \frac{20}{\sin B}$

$\sin B = \frac{20 \sin 29°}{15}$

$B = \sin^{-1}\left(\frac{20 \sin 29°}{15}\right)$

$B \approx 40.27168721$

$180° - \propto \approx 180° - 40.3°$ or $139.7°$

Solution 1

$C \approx 180° - (29° - 40.3°)$ or $110.7°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{15}{\sin 29°} \approx \frac{c}{\sin 110.7°}$

$c \approx \frac{15 \sin 110.7°}{\sin 29°}$

$c \approx 28.93721187$

Perimeter $= a + b + c$

$\approx 15 + 20 + 28.9$ or about 63.9 units

Solution 2

$C \approx 180° - (29° - 139.7°)$ or $11.3°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{15}{\sin 29°} \approx \frac{c}{\sin 11.3°}$

$c \approx \frac{15 \sin 11.3°}{\sin 29°}$

$c \approx 6.047576406$

Perimeter $= a + b + c$

$\approx 15 + 20 + 6.0$ or about 41.0 units

32. $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{13}{\sin 55°} = \frac{15}{\sin A}$

$\sin A = \frac{15 \sin 55°}{13}$

$A = \sin^{-1}\left(\frac{15 \sin 55°}{13}\right)$

$A \approx 70.93970395$

$180° - \propto \approx 180° - 70.9°$ or $109.1°$

Solution 1

$C \approx 180° - (70.9° + 55°)$ or $54.1°$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{13}{\sin 55°} \approx \frac{c}{\sin 54.1°}$

$c \approx \frac{13 \sin 54.1°}{\sin 55°}$

$c \approx 12.8489656$

Perimeter $= a + b + c$

$\approx 15 + 13 + 12.8$ or about 40.8

Solution 2

$c \approx 180° - (109.1° + 55°)$ or $15.9°$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{13}{\sin 55°} \approx \frac{c}{\sin 15.9°}$

$c \approx \frac{13 \sin 15.9°}{\sin 55°}$

$c \approx 4.35832749$

Perimeter $= a + b + c$

$\approx 15 + 13 + 4.4$ or about 32.4

$A \approx 70.9°, B = 55°, C \approx 54.1°$

33. side opposite 37° = 15 + 18 or 33
side between θ and 37° = 15 + 22 or 37
Let x = the measure of the third angle.

$$\frac{33}{\sin 37°} = \frac{37}{\sin x}$$
$$\sin x = \frac{37 \sin 37°}{33}$$
$$x = \sin^{-1}\left(\frac{37 \sin 37°}{33}\right)$$
$$x \approx 42.43569405$$
$$\theta \approx 180 - (37° + 42.4°) \text{ or about } 100.6°$$

34a. $a < b \sin A$
$a < 14 \sin 30°$
$a < 7$

34b. $a = b \sin A$
$a = 14 \sin 30°$
$a = 7$ or $a \geq 14$

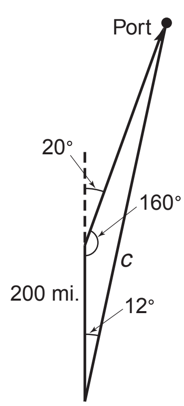
34c. $a > b \sin A$
$a > 14 \sin 30°$
$a > 7$ and $a < 14$
$7 < a < 14$

35. $\frac{184.5}{\sin 59°} = \frac{140}{\sin x}$
$$\sin x = \frac{140 \sin 59°}{184.5}$$
$$x = \sin^{-1}\left(\frac{140 \sin 59°}{184.5}\right)$$
$$x \approx 40.57365664$$
$\theta \approx 180° - (59° + 40.6°)$ or about 80.4°
$90° - 80.4° \approx 9.6°$

36a. $12° < 90°$ and $316 > 450 \sin 12°$; 2 solutions
$$\frac{316}{\sin 12°} = \frac{450}{\sin \theta}$$
$$\sin \theta = \frac{450 \sin 12°}{316}$$
$$\theta = \sin^{-1}\left(\frac{450 \sin 12°}{316}\right)$$
$$\theta \approx 17.22211674$$
$180° - \propto \approx 180° - 17.2°$ or 162.8°
turn angle $\approx 180° - 162.8°$ or 17.2°
about 17.2° east of north

36b. $\theta \approx 180° - (162.8° + 12°)$ or about 5.2°
$$\frac{x}{\sin 5.2°} = \frac{316}{\sin 12°}$$
$$x = \frac{316 \sin 5.2°}{\sin 12°}$$
$$x \approx 138.3094714$$
$$d = rt$$
$$138.3 \approx 23t$$
$6.013455278 \approx t$; about 6 hr

36c. $180° - 20° = 160°$
$180° - (160° + 12°) = 8°$
$$\frac{200}{\sin 8°} = \frac{c}{\sin 160°}$$
$$c = \frac{200 \sin 160°}{\sin 8°}$$
$$c \approx 491.5032301$$
Since $491.5 \neq 450$, the ship will not reach port.



37. Distance from satellite to center of earth is 3960 + 1240 or 5200 miles.
angle across from 5200 mi side = 45° + 90° or 135°
$$\frac{5200}{\sin 135°} = \frac{3960}{\sin x}$$
$$\sin x = \frac{3960 \sin 135°}{5200}$$
$$x = \sin^{-1}\left(\frac{3960 \sin 135°}{5200}\right)$$
$$x \approx 32.58083835$$
$\theta \approx 180° - (135° + 32.6°)$ or about 21.4°
$\frac{21.4°}{360°}$ (2 hours) ≈ 0.0689953425 hours or about 4.1 minutes

38. P turns 20(360°) or 7200° every second which equals 72° every 0.01 second.
$$\frac{PQ}{\sin O} = \frac{OP}{\sin Q}$$
$$\frac{15}{\sin 72°} = \frac{5}{\sin Q}$$
$$\sin Q = \frac{5 \sin 72°}{15}$$
$$Q = \sin^{-1}\left(\frac{5 \sin 72°}{15}\right)$$
$$Q \approx 18.48273235$$
$m\angle P \approx 180° - (72° + 18.5°)$ or about 89.5°
$$\frac{QO}{\sin P} = \frac{PQ}{\sin O}$$
$$\frac{QO}{\sin 89.5°} \approx \frac{15}{\sin 72°}$$
$$QO \approx \frac{15 \sin 89.5°}{\sin 72°}$$
$$QO \approx 15.77133282$$
$QO - 5 \approx 15.8 - 5$ or about 10.8 cm

39a.
$$b < c \sin B$$
$$12 < 17 \sin B$$
$$\frac{12}{17} < \sin B$$
$$\sin^{-1} \frac{12}{17} < B$$
$$44.90087216 < B$$
$$B > 44.9°$$

39b.
$$b = c \sin B$$
$$12 = 17 \sin B$$
$$\frac{12}{17} = \sin B$$
$$\sin^{-1} \frac{12}{17} = B$$
$$44.90087216 \approx B$$
$$B \approx 44.9°$$

39c.
$$b > c \sin B$$
$$12 > 17 \sin B$$
$$\frac{12}{17} > \sin B$$
$$\sin^{-1} \frac{12}{17} > B$$
$$44.90087216 > B$$
$$B < 44.9°$$

40. Area of rhombus = 2(Area of triangle)
triangle:
$$K = \frac{1}{2}bc \sin A$$
$$K = \frac{1}{2}(24)(24) \sin 32°$$
$$K \approx 152.6167481$$
rhombus:
$A \approx 2(152.6)$ or about 305.2 in^2

41. $\tan 22° = \frac{75}{x}$

$x = \frac{75}{\tan 22°}$

$x \approx 185.6$ m

42. $3; \frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & -4 & 13 & -6 \\ & & 2 & -1 & 6 \\ \hline & 4 & -2 & 12 & 0 \end{array}$$

$4x^2 - 2x + 12 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(4)(12)}}{2(4)}$

$x = \frac{2 \pm \sqrt{-188}}{8}$

$x = \frac{2 \pm 2i\sqrt{47}}{8}$

$x = \frac{1 \pm i\sqrt{47}}{4}$

43. no

$$\frac{\frac{3x}{x-1} + 1}{3\left(\frac{3x}{x-1}\right)} = \frac{\frac{3x}{x-1} + \frac{x-1}{x-1}}{\frac{9x}{x-1}} = \frac{\frac{4x-1}{x-1}}{\frac{9x}{x-1}} = \frac{4x-1}{9x} \neq x$$

44. $5x - 2y = 9$ $\quad$ $y = 3x - 1$

$5x - 2(3x - 1) = 9$ $\quad$ $y = 3(-7) - 1$

$5x - 6x + 2 = 9$ $\quad$ $y = -22$

$x = -7$

$(-7, -22)$

45. $-2x + 5y = 7$

$y = \frac{2}{5}x + \frac{7}{5}$

perpendicular slope: $-\frac{5}{2}$

$y - 4 = -\frac{5}{2}(x - (-6))$

$y - 4 = -\frac{5}{2}x - 15$

$2y - 8 = -5x - 30$

$5x + 2y = -22$

46. Perimeter of $XYZ = 4 + 8 + 9$ or 21

length of $\overline{AB} = \frac{1}{3}$ of perimeter

$= \frac{1}{3}(21)$ or 7

The answer is 7.

5-8 The Law of Cosines

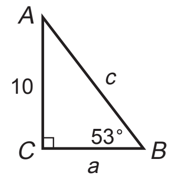
Pages 330–331 Check for Understanding

1. The Law of Cosines is needed to solve a triangle if the measures of all three sides or the measures of two sides and the included angle are given.

2. Sample answer: 1 in., 2 in., 4 in.

3. If the included angle measures 90°, the equation becomes $c^2 = a^2 + b^2 - 2ab \cos C$. Since $\cos 90° = 0$, $c^2 = a^2 + b^2 - 2ab(0)$ or $c^2 = a^2 + b^2$.

4. Sample answers:



$A = 180° - (90° + 53°)$ or 37°

$\sin B = \frac{b}{c}$

$\sin 53° = \frac{10}{c}$

$c = \frac{10}{\sin 53°}$

$c \approx 12.52135658$

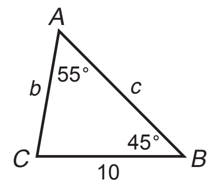
$\tan B = \frac{b}{a}$

$\tan 53° = \frac{10}{a}$

$a = \frac{10}{\tan 53°}$

$a \approx 7.535540501$

$A = 37°, a \approx 7.5, c \approx 12.5$



$C = 180° - (5.5° + 45°)$ or 80°

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 45°} = \frac{10}{\sin 55°}$

$b = \frac{10 \sin 45°}{\sin 55°}$

$b \approx 8.6321799$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 80°} = \frac{10}{\sin 55°}$

$c = \frac{10 \sin 80°}{\sin 55°}$

$c \approx 12.0222828$

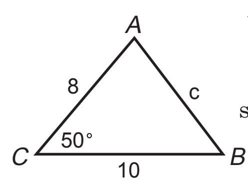
$C = 80°, b \approx 8.6, c \approx 12.0$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 10^2 + 8^2 - 2(10)(8) \cos 50°$

$c^2 \approx 61.15398245$

$c \approx 7.820101179$



$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{10}{\sin A} \approx \frac{7.8}{\sin 50°}$

$\sin A \approx \frac{10 \sin 50°}{7.8}$

$A \approx \sin^{-1}\left(\frac{10 \sin 50°}{7.8}\right)$

$A \approx 78.4024367$

$B \approx 180° - (78.4° + 50°)$ or 51.6°

$A \approx 78.4°, B \approx 51.6°, c \approx 7.8$

5. $a^2 = b^2 + c^2 - 2bc \cos A$

$32^2 = 38^2 + 46^2 - 2(38)(46) \cos A$

$\frac{32^2 - 38^2 - 46^2}{-2(38)(46)} = \cos A$

$\cos^{-1}\left(\frac{32^2 - 38^2 - 46^2}{-2(38)(46)}\right) = A$

$43.49782861 \approx A$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{32}{\sin 43.5°} \approx \frac{38}{\sin B}$

$\sin B \approx \frac{38 \sin 43.5°}{32}$

$B \approx \sin^{-1}\left(\frac{38 \sin 43.5°}{32}\right)$

$B \approx 54.8$

$C \approx 180° - (43.5° + 54.8°)$ or 81.7°

$A = 43.5°, B = 54.8°, C = 81.7°$

6. $c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 25^2 + 30^2 - 2(25)(30) \cos 160°$

$c^2 \approx 2934.538931$

$c \approx 54.1713848$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{54.2}{\sin 160°} \approx \frac{25}{\sin A}$

$\sin A \approx \frac{25 \sin 160°}{54.2}$

$A \approx \sin^{-1}\left(\frac{25 \sin 160°}{54.2}\right)$

$A \approx 9.1$

$B \approx 180° - (9.1° + 160°)$ or $10.9°$

$A = 9.1°, B = 10.9°, c = 54.2$

7. The angle with greatest measure is across from the longest side.

$21^2 = 18^2 + 14^2 - 2(18)(14) \cos \theta$

$\frac{21^2 - 18^2 - 14^2}{-2(18)(14)} = \cos \theta$

$\cos^{-1}\left(\frac{21^2 - 18^2 - 14^2}{-2(18)(14)}\right) = \theta$

$81.0 \approx \theta$

about 81.0°

8. $s = \frac{1}{2}(2 + 7 + 8) = 8.5$

$K = \sqrt{8.5(8.5 - 2)(8.5 - 7)(8.5 - 8)}$

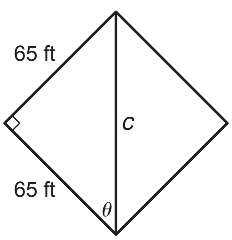
≈ 6.4 units^2

9. $s = \frac{1}{2}(25 + 13 + 17) = 27.5$

$K = \sqrt{27.5(27.5 - 25)(27.5 - 13)(27.5 - 17)}$

≈ 102.3 units^2

10.



$a^2 + b^2 = c^2$

$65^2 + 65^2 = c^2$

$8450 = c^2$

$91.92388155 \approx c$

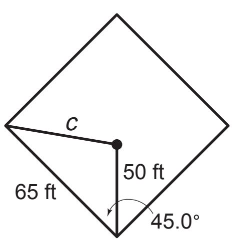
$a^2 = b^2 + c^2 - 2bc \cos \theta$

$65^2 \approx 65^2 + 91.9^2 - 2(65)(91.9) \cos \theta$

$\frac{65^2 - 65^2 - 91.9^2}{-2(65)(91.9)} \approx \cos \theta$

$\cos^{-1}\left(\frac{65^2 - 65^2 - 91.9^2}{-2(65)(91.9)}\right) \approx \theta$

$45.01488334 \approx \theta$



$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 \approx 65^2 + 50^2 - 2(65)(50) \cos 45.0°$

$c^2 \approx 2128.805922$

$c \approx 46.1$ ft

Pages 331–332 Exercises

11. $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 7^2 + 10^2 - 2(7)(10) \cos 51°$

$a^2 \approx 60.89514525$

$a \approx 7.803534151$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{7.8}{\sin 51°} \approx \frac{7}{\sin B}$

$\sin B \approx \frac{7 \sin 51°}{7.8}$

$B \approx \sin^{-1}\left(\frac{7 \sin 51°}{7.8}\right)$

$B \approx 44.22186872$

$C \approx 180° - (51° + 44.2°)$ or $84.8°$

$B = 44.2°, C = 84.8°, a = 7.8$

12. $c^2 = a^2 + b^2 - 2ab \cos C$

$7^2 = 5^2 + 6^2 - 2(5)(6) \cos C$

$\frac{7^2 - 5^2 - 6^2}{-2(5)(6)} = \cos C$

$\cos^{-1}\left(\frac{7^2 - 5^2 - 6^2}{-2(5)(6)}\right) = C$

$78.46304097 \approx C$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{5}{\sin A} \approx \frac{7}{\sin 78.5°}$

$\sin A \approx \frac{5 \sin 78.5°}{7}$

$A \approx \sin^{-1}\left(\frac{5 \sin 78.5°}{7}\right)$

$A \approx 44.42268919$

$B \approx 180° - (44.4° + 78.5°)$ or $57.1°$

$A = 44.4°, B = 57.1°, C = 78.5°$

13. $c^2 = a^2 + b^2 - 2ab \cos C$

$7^2 = 4^2 + 5^2 - 2(4)(5) \cos C$

$\frac{7^2 - 4^2 - 5^2}{-2(4)(5)} = \cos C$

$\cos^{-1}\left(\frac{7^2 - 4^2 - 5^2}{-2(4)(5)}\right) = C$

$101.536959 \approx C$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{4}{\sin A} \approx \frac{7}{\sin 101.5°}$

$\sin A \approx \frac{4 \sin 101.5°}{7}$

$A \approx \sin^{-1}\left(\frac{4 \sin 101.5°}{7}\right)$

$A \approx 34.05282227$

$B \approx 180° - (34.1° + 101.5°)$ or $44.4°$

$A = 34.1°, B = 44.4°, C = 101.5°$

14. $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 16^2 + 12^2 - 2(16)(12) \cos 63°$

$b^2 \approx 225.6676481$

$b \approx 15.02223845$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{16}{\sin A} \approx \frac{15.0}{\sin 63°}$

$\sin A \approx \frac{16 \sin 63°}{15.0}$

$A \approx \sin^{-1}\left(\frac{16 \sin 63°}{15.0}\right)$

$A \approx 71.62084388$

$C \approx 180° - (71.6° + 63°)$ or $45.4°$

$A = 71.6°, C = 45.4°, b = 15.0$

15. $b^2 = a^2 + c^2 - 2ac\cos B$

$13.7^2 = 11.4^2 + 12.2^2 - 2(11.4)(12.2)\cos B$

$\frac{13.7^2 - 11.4^2 - 12.2^2}{-2(11.4)(12.2)} = \cos B$

$\cos^{-1}\left(\frac{13.7^2 - 11.4^2 - 12.2^2}{-2(11.4)(12.2)}\right) = B$

$70.8801474 \approx B$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{11.4}{\sin A} \approx \frac{13.7}{\sin 70.9°}$

$\sin A \approx \frac{11.4 \sin 70.9°}{13.7}$

$A \approx \sin^{-1}\left(\frac{11.4 \sin 70.9°}{13.7}\right)$

$A \approx 51.84180107$

$C \approx 180° - (51.8° + 70.9°)$ or $57.3°$

$A = 51.8°, B = 70.9°, C = 57.3°$

16. $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 21.5^2 + 13^2 - 2(21.5)(13)\cos 79.3°$

$c^2 \approx 527.462362$

$c \approx 22.96654876$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{21.5}{\sin A} \approx \frac{23.0}{\sin 79.3°}$

$\sin A \approx \frac{21.5 \sin 79.3°}{23.0}$

$A \approx \sin^{-1}\left(\frac{21.5 \sin 79.3°}{23.0}\right)$

$A \approx 66.90667662$

$B \approx 180° - (66.9° - 79.3°)$ or $33.8°$

$A = 66.9°, B = 33.8°, c = 23.0$

17. $14.9^2 = 23.8^2 + 36.9^2 - 2(23.8)(36.9)\cos\theta$

$\frac{14.9^2 - 23.8^2 - 36.9^2}{-2(23.8)(36.9)} = \cos\theta$

$\cos^{-1}\left(\frac{14.9^2 - 23.8^2 - 36.9^2}{-2(23.8)(36.9)}\right) = \theta$

$13.75878964 \approx \theta$

about 13.8°

18. $d_1{}^2 = 40^2 + 60^2 - 2(40)(60)\cos 132°$

$d_1{}^2 \approx 8411.826911$

$d_1 \approx 91.71601229$

$180° - 132° = 48°$

$d_2{}^2 = 40^2 + 60^2 - 2(40)(60)\cos 48°$

$d_2{}^2 \approx 1988.173089$

$d_2 \approx 44.58893461$

about 91.7 cm and 44.6 cm

19. $s = \frac{1}{2}(4 + 6 + 8) = 9$

$K = \sqrt{9(9 - 4)(9 - 6)(9 - 8)}$

≈ 11.6 units^2

20. $s = \frac{1}{2}(17 + 13 + 19) = 24.5$

$K = \sqrt{24.5(24.5 - 17)(24.5 - 13)(24.5 - 19)}$

≈ 107.8 units^2

21. $s = \frac{1}{2}(20 + 30 + 40) = 45$

$K = \sqrt{45(45 - 20)(45 - 30)(45 - 40)}$

≈ 290.5 units^2

22. $s = \frac{1}{2}(33 + 51 + 42) = 63$

$K = \sqrt{63(63 - 33)(63 - 51)(63 - 42)}$

≈ 690.1 units^2

23. $s = \frac{1}{2}(174 + 138 + 188) = 250$

$K = \sqrt{250 \cdot 76 \cdot 112 \cdot 62}$

$\approx 11{,}486.3$ units^2

24. $s = \frac{1}{2}(11.5 + 13.7 + 12.2) = 18.7$

$K = \sqrt{187(18.7 - 11.5)(18.7 - 13.7)(18.7 - 12.2)}$

≈ 66.1 units^2

25a. $d^2 = 30^2 + 48^2 - 2(30)(48)\cos 120°$

$d^2 = 4644$

$d \approx 68.1$ in.

25b. Area of parallelogram = 2(Area of triangle)

$K = \frac{1}{2}(30)(48)\sin 120°$

$K \approx 623.5382907$

$2K \approx 2(623.5382907)$ or about 1247.1 in^2

26a. $s = \frac{1}{2}(15 + 15 + 24.6) = 27.3$

$K = \sqrt{27.3(27.3 - 15)(27.3 - 15)(27.3 - 24.6)}$

≈ 105.6

Area of rhombus $\approx 2(105.6) \approx 211.2$ cm^2

26b. $24.6^2 = 15^2 + 15^2 - 2(15)(15)\cos\theta$

$\frac{24.6^2 - 15^2 - 15^2}{-2(15)(15)} = \cos\theta$

$\cos^{-1}\left(\frac{24.6^2 - 15^2 - 15^2}{-2(15)(15)}\right) = \theta$

$110.1695875 \approx \theta$

$180° - 110.2° = 69.8°$

about 110.2°, 69.8°, 110.2°, 69.8°

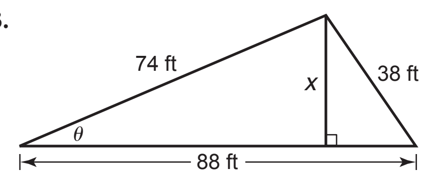
27. The angle opposite the missing side = 45°.

$x^2 = 400^2 + 90^2 - 2(400)(90)\cos 45°$

$x^2 \approx 117{,}188.3118$

$x \approx 342.3$ ft

28.



$38^2 = 74^2 + 88^2 - 2(74)(88)\cos\theta$

$\frac{38^2 - 74^2 - 88^2}{-2(74)(88)} = \cos\theta$

$\cos^{-1}\left(\frac{38^2 - 74^2 - 88^2}{-2(74)(88)}\right) = \theta$

$25.28734695 \approx \theta$

$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$

$\sin 25.3° \approx \frac{x}{74}$

$31.60970664 \approx x$

about 31.6 ft

29a. $x^2 = 100^2 + 220^2 - 2(100)(220)\cos 10°$

$x^2 \approx 15{,}068.45887$

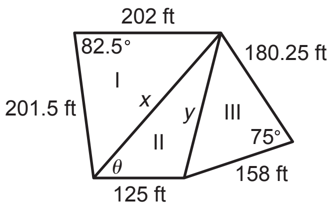
$x \approx 122.7536511$

about 122.8 mi

29b. $(100 + 122.7536511) - 220 \approx 2.7536511$

about 2.8 mi

30.



I: $K = \frac{1}{2}(201.5)(202)\sin 82.5°$

$K \approx 20{,}177.3901$

II: $x^2 = 201.5^2 + 202^2 - 2(201.5)(202)\cos 82.5°$

$x^2 \approx 70{,}780.6348$

$x \approx 266.046302$

$y^2 = 158^2 + 180.25^2 - 2(158)(180.25)\cos 75°$

$y^2 \approx 42.711.98851$

$y \approx 206.6687894$

$206.7^2 \approx 266.0^2 + 125^2 - 2(266.0)(125)\cos\theta$

$\frac{206.7^2 - 266.0^2 - 125^2}{-2(266.0)(125)} \approx \cos\theta$

$\cos^{-1}\left(\frac{206.7^2 - 266.0^2 - 125^2}{-2(266.0)(125)}\right) \approx \theta$

$48.93361962 \approx \theta$

$K \approx \frac{1}{2}(266.0)(125)\sin 48.9°$

$K \approx 12{,}536.58384$

III: $K = \frac{1}{2}(180.25)(158)\sin 75°$

$K \approx 13{,}754.54228$

Area of pentagon = I + II + III

$\approx 20{,}177.4 + 12{,}536.6 + 13{,}754.5$

$\approx 46{,}468.5$ ft

31. I: $24^2 = 35^2 + 40^2 - 2(35)(40)\cos\theta$

$\frac{24^2 - 35^2 - 40^2}{-2(35)(40)} = \cos\theta$

$\cos^{-1}\left(\frac{24^2 - 35^2 - 40^2}{-2(35)(40)}\right) = \theta$

$36.56185036 \approx \theta$

II: $24^2 = 30^2 + 20^2 - 2(30)(20)\cos\theta$

$\frac{24^2 - 30^2 - 20^2}{-2(30)(20)} = \cos\theta$

$\cos^{-1}\left(\frac{24^2 - 30^2 - 20^2}{-2(30)(20)}\right) = \theta$

$52.89099505 \approx \theta$

the player 30 ft and 20 ft from the posts

32a. $\sin 6° = \frac{20{,}000}{x}$

$x = \frac{20{,}000}{\sin 6°}$

$x \approx 191{,}335.4$ ft

32b. $\sin 3° = \frac{15{,}000}{y}$

$y = \frac{15{,}000}{\sin 3°}$

$y \approx 286{,}609.8$ ft

32c. $6° - 3° = 3°$

$d^2 \approx 191{,}335.4^2 + 286{,}609.8^2 - 2(191{,}335.4)(286{,}609.8)\cos 3°$

$d^2 \approx 9{,}227{,}519{,}077$

$d \approx 96{,}060.0$ ft

33. Since $63.2° < 90°$, consider Case I.

$b \sin A = 18 \sin 63.2°$

$b \sin A \approx 16.06654473$

$17 > 16.1$; 2 solutions

34. $\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$

$\tan\theta = \frac{570}{700}$

$\theta = \tan^{-1}\frac{570}{700}$

$\theta \approx 39.2°$

35. $-775° + 2(360°) = -55°$

reference angle = $|-55°|$ or $55°$

36.

3	1	−7	−k	6
		3	−12	−36 − 3k
	1	−4	−12 − k	−30 − 3k

$-30 - 3k = 0$

$k = -10$

37. $m = \frac{5t - t}{5t - 2t}$

$m = \frac{4t}{3t}$ or $\frac{4}{3}$

38. $\left(\frac{2x^2}{y}\right)^3 = \frac{2^3x^6}{y^3}$

$= \frac{8x^6}{y^3}$

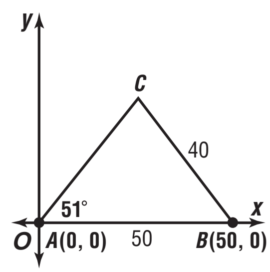
The correct choice is A.

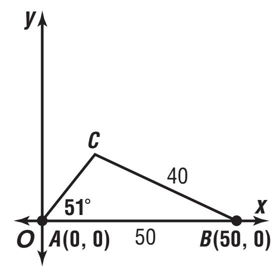
5-8B Graphing Calculator Exploration: Solving Triangles

Page 334

1. AB ≈ 12.1, $B \approx 25.5°$, $C \approx 119.5°$

2.





Find y using $\overline{AC}$.

$y = (\tan 51°)x$

Find y using $\overline{BC}$.

$\frac{40}{\sin 51°} = \frac{50}{\sin C}$

$\sin C = \frac{50 \sin 51°}{40}$

$C = \sin^{-1}\left(\frac{50 \sin 51°}{40}\right)$

$C \approx 76.27180414$

$B \approx 80 - 51 - 76.27180414$

≈ 52.72819586

$\tan(180 - 52.72819586) \approx \frac{y}{x - 50}$

$(x - 50)\tan(127.2718041) \approx y$

Set the two values of y equal to each other.

$(\tan 51°)x \approx (x - 50), \tan 127.2718041°$

$(\tan 51°)x \approx x(\tan 127.2718041° - 50(\tan 127.2718041°)$

$x \approx \frac{-50(\tan 127.2718041°)}{\tan 51° - \tan 127.271804°}$

$x \approx 25.77612538$

$y \approx (\tan 51°)(25.77612538) \approx 31.83086394$

C could also equal 180 − 76.27180414 or 103.728195°
$B \approx 180 - 51 - 103.7281959°$
≈ 25.2718041
$\tan(180 - 25.2718041) \approx \frac{y}{x - 50}$
$(x - 50)\tan 154.7281959° \approx y$
Set the two values of y equal to each other.
$(\tan 51°)\, x \approx (x - 50)\tan 154.7281959°$
$(\tan 51°)\, x \approx x(\tan 154.7281959°) - 50(\tan 154.7281959°)$
$x \approx \frac{-50(\tan 154.7281959°)}{\tan 51° - \tan 154.7281959°}$
$x \approx 13.82829048$
$y \approx (\tan 51°)(13.82828048)$
≈ 17.07651659
$B \approx 52.7°$, $C \approx 76.3°$, $b \approx 40.9$; $B \approx 25.3°$, $C \approx 103.7°$, $b \approx 220$

3. Law of Cosines

4. Sample answer: put vertex A at the origin and vertex C at (3, 0).

Chapter 5 Study Guide and Assessment

Page 335 Understanding and Using the Vocabulary

1. false; depression
2. false; arcosine
3. true
4. false; adjacent to
5. true
6. false; coterminal
7. true
8. false; Law of Cosines
9. false; terminal side
10. true

Pages 336–338 Skills and Concepts

11. $57.15° = 57° + (0.15 \cdot 60)'$
$= 57° + 9'$
$57°\,9'$

12. $-17.125° = -(17° + (0.125 \cdot 60)')$
$= -(17° + 7.5')$
$= -(17° + 7' + (0.5 \cdot 60)'')$
$= -(17° + 7' + 30'')$
$-17°\,7'\,30''$

13. $\frac{860°}{360°} \approx 2.39$
$\propto + 360(2)° = 860°$
$\propto + 720° = 860°$
$\propto = 140°$; II

14. $\frac{1146°}{360°} \approx 3.18$
$\propto + 360(3)° = 1146°$
$\propto + 1080° = 1146°$
$\propto = 66°$; I

15. $\frac{-156°}{360°} \approx -0.43$
$\propto + 360(-1)° = -156°$
$\propto - 360° = -156°$
$\propto = 204°$; III

16. $\frac{998°}{360°} \approx 2.77$
$\propto + 360(2)° = 998°$
$\propto + 720° = 998°$
$\propto = 278°$; IV

17. $\frac{-300°}{360°} \approx -0.83$
$\propto + 360(-1)° = -300°$
$\propto - 360° = -300°$
$\propto = 60°$; I

18. $\frac{1072°}{360°} \approx 2.98$
$\propto + 360(2)° = 1072°$
$\propto + 720° = 1072°$
$\propto = 352°$; IV

19. $\frac{654°}{360°} \approx 1.82$
$\propto + 360(1)° = 654°$
$\propto + 360° = 654°$
$\propto = 294°$; IV

20. $\frac{-832°}{360°} \approx -2.31$
$\propto + 360(-2)° = -832°$
$\propto - 720° = -832°$
$\propto = -112°$
$360° - 112° = 248°$; III

21. $-284°$ has terminal side in first quadrant.
$360° - 284° = 76°$

22. $\frac{592°}{360°} \approx 1.64$
$\propto + 360(1)° = 592°$
$\propto + 360° = 592°$
$\propto = 232°$
terminal side in third quadrant
$232° - 180° = 52°$

23. $(BC)^2 + (AC)^2 = (AB)^2$
$15^2 + 9^2 = (AB)^2$
$306 = (AB)^2$
$\sqrt{306} = AB$
$3\sqrt{34} = AB$
$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\quad\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$
$\sin A = \frac{15}{3\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$ $\quad\cos A = \frac{9}{3\sqrt{34}}$ or $\frac{3\sqrt{34}}{34}$
$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$
$\tan A = \frac{15}{9}$ or $\frac{5}{3}$

24. $(PM)^2 + (PN)^2 = (MN)^2$
$8^2 + 12^2 = (MN)^2$
$208 = (MN)^2$
$\sqrt{208} = MN$
$4\sqrt{13} = MN$

$\sin M = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos M = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\sin M = \frac{12}{4\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$ $\cos M = \frac{8}{4\sqrt{13}}$ or $\frac{2\sqrt{13}}{13}$

$\tan M = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc M = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan M = \frac{12}{8}$ or $\frac{3}{2}$ $\csc M = \frac{4\sqrt{13}}{12}$ or $\frac{\sqrt{13}}{3}$

$\sec M = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot M = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec M = \frac{4\sqrt{13}}{8}$ or $\frac{\sqrt{13}}{2}$ $\cot M = \frac{8}{12}$ or $\frac{2}{3}$

25. $(MP)^2 + (PN)^2 = (MN)^2$
$(MP)^2 + 10^2 = 12^2$
$(MP)^2 = 44$
$MP = \sqrt{44}$ or $2\sqrt{11}$

$\sin M = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos M = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\sin M = \frac{10}{12}$ or $\frac{5}{6}$ $\cos M = \frac{2\sqrt{11}}{12}$ or $\frac{\sqrt{11}}{6}$

$\tan M = \frac{\text{side opposite}}{\text{side adjacent}}$ $\csc M = \frac{\text{hypotenuse}}{\text{side opposite}}$

$\tan M = \frac{10}{2\sqrt{11}}$ or $\frac{5\sqrt{11}}{11}$ $\csc M = \frac{12}{10}$ or $\frac{6}{5}$

$\sec M = \frac{\text{hypotenuse}}{\text{side adjacent}}$ $\cot M = \frac{\text{side adjacent}}{\text{side opposite}}$

$\sec M = \frac{12}{2\sqrt{11}}$ or $\frac{6\sqrt{11}}{11}$ $\cot M = \frac{2\sqrt{11}}{10}$ or $\frac{\sqrt{11}}{5}$

26. $\sec \theta = \frac{1}{\cos \theta}$

$\cos \theta = \frac{1}{\sec\theta}$

$\cos \theta = \frac{1}{\frac{7}{5}}$ or $\frac{5}{7}$

27. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{3^2 + 3^2}$
$r = \sqrt{18}$ or $3\sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{3}{3\sqrt{2}}$ $\cos \theta = \frac{3}{3\sqrt{2}}$ $\tan \theta = \frac{3}{3}$ or 1

$\sin \theta = \frac{\sqrt{2}}{2}$ $\cos \theta = \frac{\sqrt{2}}{2}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{3\sqrt{2}}{3}$ or $\sqrt{2}$ $\sec \theta = \frac{3\sqrt{2}}{3}$ or $\sqrt{2}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{3}{3}$ or 1

28. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-5)^2 + 12^2}$
$r = \sqrt{169}$ or 13

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{12}{13}$ $\cos \theta = \frac{-5}{13}$ or $-\frac{5}{13}$ $\tan \theta = \frac{12}{-5}$ or $-\frac{12}{5}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{13}{12}$ $\sec \theta = \frac{13}{-5}$ or $-\frac{13}{5}$ $\cot \theta = \frac{-5}{12}$ or $-\frac{5}{12}$

29. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{8^2 + (-2)^2}$
$r = \sqrt{68}$ or $2\sqrt{17}$

$\sin \theta = \frac{y}{x}$ $\cos \theta = \frac{x}{y}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-2}{2\sqrt{17}}$ $\cos \theta = \frac{8}{2\sqrt{17}}$ $\tan \theta = \frac{-2}{8}$ or $-\frac{1}{4}$

$\sin \theta = -\frac{\sqrt{17}}{17}$ $\cos \theta = \frac{4\sqrt{17}}{17}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{2\sqrt{17}}{-2}$ or $-\sqrt{17}$ $\sec \theta = \frac{2\sqrt{17}}{8}$ or $\frac{\sqrt{17}}{4}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{8}{-2}$ or -4

30. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-2)^2 + 0^2}$
$r = \sqrt{4}$ or 2

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{0}{2}$ or 0 $\cos \theta = \frac{-2}{2}$ or -1 $\tan \theta = \frac{0}{-2}$ or 0

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{2}{0}$ $\sec \theta = \frac{2}{-2}$ or -1 $\cot \theta = \frac{-2}{0}$

undefined undefined

31. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{4^2 + 5^2}$
$r = \sqrt{41}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{5}{\sqrt{41}}$ $\cos \theta = \frac{4}{\sqrt{41}}$ $\tan \theta = \frac{5}{4}$

$\sin \theta = \frac{5\sqrt{41}}{41}$ $\cos \theta = \frac{4\sqrt{41}}{41}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{\sqrt{41}}{5}$ $\sec \theta = \frac{\sqrt{41}}{4}$ $\cot \theta = \frac{4}{5}$

32. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-5)^2 + (-9)^2}$
$r = \sqrt{106}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-9}{\sqrt{106}}$ $\cos \theta = \frac{-5}{\sqrt{106}}$ $\tan \theta = \frac{-9}{-5}$ or $\frac{9}{5}$

$\sin \theta = -\frac{9\sqrt{106}}{106}$ $\cos \theta = -\frac{5\sqrt{106}}{106}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$

$\csc \theta = \frac{\sqrt{106}}{-9}$ or $-\frac{\sqrt{106}}{9}$ $\sec \theta = \frac{\sqrt{106}}{-5}$ or $-\frac{\sqrt{106}}{5}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{-5}{-9}$ or $\frac{5}{9}$

33. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{(-4)^2 + 4^2}$
$r = \sqrt{32}$ or $4\sqrt{2}$

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{4}{4\sqrt{2}}$ $\cos \theta = \frac{-4}{4\sqrt{2}}$ $\tan \theta = \frac{4}{-4}$ or -1

$\sin \theta = \frac{\sqrt{2}}{2}$ $\cos \theta = -\frac{\sqrt{2}}{2}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

$\csc \theta = \frac{4\sqrt{2}}{4}$ $\sec \theta = \frac{4\sqrt{2}}{-4}$ $\cot \theta = \frac{-4}{4}$ or -1

$\csc \theta = \sqrt{2}$ $\sec \theta = -\sqrt{2}$

34. $r = \sqrt{x^2 + y^2}$
$r = \sqrt{5^2 + 0^2}$
$r = \sqrt{25}$ or 5

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{0}{5}$ or 0 $\cos\theta = \frac{5}{5}$ or 1 $\tan\theta = \frac{0}{5}$ or 0

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{5}{0}$ $\sec\theta = \frac{5}{5}$ or 1 $\cot\theta = \frac{5}{0}$

undefined undefined

35. $\cos\theta = \frac{x}{r}$ $r^2 = x^2 + y^2$
$\cos\theta = -\frac{3}{8}$ $8^2 = (-3)^2 + y^2$
$x = -3, r = 8$ $55 = y^2$
$\pm\sqrt{55} = y$
Quadrant 11, so $y = \sqrt{55}$

$\sin\theta = \frac{y}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{\sqrt{55}}{8}$ $\tan\theta = \frac{\sqrt{55}}{-3}$ or $-\frac{\sqrt{55}}{3}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{8}{\sqrt{55}}$ $\sec\theta = \frac{8}{-3}$ or $-\frac{8}{3}$ $\cot\theta = \frac{-3}{\sqrt{55}}$

$\csc\theta = \frac{8\sqrt{55}}{55}$ $\cot\theta = -\frac{3\sqrt{55}}{55}$

36. $\tan\theta = \frac{y}{x}$ $r^2 = x^2 + y^2$
$\tan\theta = 3$; Quadrant III $r^2 = (-1)^2 + (-3)^2$
$y = -3, x = -1$ $r^2 = 10$
$r = \sqrt{10}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\csc\theta = \frac{r}{x}$

$\sin\theta = \frac{-3}{\sqrt{10}}$ $\cos\theta = \frac{-1}{\sqrt{10}}$ $\csc\theta = \frac{\sqrt{10}}{-3}$ or $-\frac{\sqrt{10}}{3}$

$\sin\theta = -\frac{3\sqrt{10}}{10}$ $\cos\theta = -\frac{\sqrt{10}}{10}$

$\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

$\sec\theta = \frac{\sqrt{10}}{-1}$ or $-\sqrt{10}$ $\cot\theta = \frac{-1}{-3}$ or $\frac{1}{3}$

37. $\sin B = \frac{b}{c}$
$\sin 42° = \frac{b}{15}$
$15 \sin 42° = b$
$10.0 \approx b$

38. $\sin A = \frac{a}{c}$
$\sin 38° = \frac{24}{c}$
$c \sin 38° = 24$
$c = \frac{24}{\sin 38°}$
$c \approx 39.0$

39. $\tan B = \frac{b}{a}$
$\tan 67° = \frac{24}{a}$
$a \tan 67° = 24$
$a = \frac{24}{\tan 67°}$
$a \approx 10.2$

40. 30°, 210°

41. 180°

42. $A + 49° = 90°$
$A = 41°$

$\tan B = \frac{b}{a}$ $\cos B = \frac{a}{c}$
$\tan 49° = \frac{b}{16}$ $\cos 49° = \frac{16}{c}$
$16 \tan 49° = b$ $c \cos 49° = 16$
$18.4 \approx b$ $c = \frac{16}{\cos 49°}$
$c \approx 24.4$

$A = 41°, b = 18.4, c = 24.4$

43. $a^2 + b^2 = c^2$ $\cos A = \frac{b}{c}$
$a^2 + 15^2 = 20^2$ $\cos A = \frac{15}{20}$
$a = \sqrt{175}$
$a \approx 13.2$ $A = \cos^{-1}\frac{15}{20}$
$A \approx 41.4°$

$41.40962211° + B \approx 90°$
$B \approx 48.6°$
$a = 13.2, A = 41.4°, B = 48.6°$

44. $64° + B = 90°$
$B = 26°$

$\sin A = \frac{a}{c}$ $\cos A = \frac{b}{c}$
$\sin 64° = \frac{a}{28}$ $\cos 64° = \frac{b}{28}$
$28 \sin 64° = a$ $28 \cos 64° = b$
$25.2 \approx a$ $12.3 \approx b$
$B = 26°, a = 25.2, b = 12.3$

45. $A = 180° - (70° + 58°)$ or 52°

$\frac{b}{\sin B} = \frac{a}{\sin A}$ $\frac{c}{\sin C} = \frac{a}{\sin A}$
$\frac{b}{\sin 70°} = \frac{84}{\sin 52°}$ $\frac{c}{\sin 58°} = \frac{84}{\sin 52°}$
$b = \frac{84 \sin 70°}{\sin 52°}$ $c = \frac{84 \sin 58°}{\sin 52°}$
$b \approx 100.1689124$ $c \approx 90.39983243$
$A = 52°, b = 100.2, c = 90.4$

46. $A = 180° - (57° + 49°)$ or 74°

$\frac{a}{\sin A} = \frac{c}{\sin C}$ $\frac{b}{\sin B} = \frac{c}{\sin C}$
$\frac{a}{\sin 74°} = \frac{8}{\sin 49°}$ $\frac{b}{\sin 57°} = \frac{8}{\sin 49°}$
$a = \frac{8 \sin 74°}{\sin 49°}$ $b = \frac{8 \sin 57°}{\sin 49°}$
$a \approx 10.1891739$ $b \approx 8.889995197$
$A = 74°, a = 10.2, b = 8.9$

47. $B = 180° - (20° + 64°)$ or 96°
$K = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$
$K = \frac{1}{2}(19)^2 \frac{\sin 96° \sin 64°}{\sin 20°}$
$K \approx 471.7$ units^2

48. $C = 180° - (56° + 78°)$ or 46°
$K = \frac{1}{2}b^2 \frac{\sin A \sin C}{\sin B}$
$K = \frac{1}{2}(24)^2 \frac{\sin 56° \sin 46°}{\sin 78°}$
$K \approx 175.6$ units^2

49. $K = \frac{1}{2} bc \sin A$
$K = \frac{1}{2}(65.5)(89.4) \sin 58.2°$
$K \approx 2488.4$ units^2

50. $K = \frac{1}{2} ac \sin B$
$K = \frac{1}{2}(18.4)(6.7) \sin 22.6°$
$K \approx 23.7$ units^2

51. Since 38.7° < 90°, consider Case I.
$c \sin A = 203 \sin 38.7°$
$c \sin A \approx 126.9242592$
172 > 126.9; 2 solutions
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{172}{\sin 38.7°} = \frac{203}{\sin C}$$
$$\sin C = \frac{203 \sin 38.7°}{172}$$
$$C = \sin^{-1}\left(\frac{203 \sin 38.7°}{172}\right)$$
$$C \approx 47.55552829$$
$180° - \propto \approx 180° - 47.6°$ or 132.4°
Solution 1
$B \approx 180° - (38.7° + 47.6°)$ or 93.7°
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 937°} \approx \frac{172}{\sin 38.7°}$$
$$b \approx \frac{172 \sin 93.7°}{\sin 38.7°}$$
$$b \approx 274.5059341$$
Solution 2
$B \approx (180° - (38.7° + 132.4°)$ or 8.9°
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 8.9°} \approx \frac{172}{\sin 38.7°}$$
$$b \approx \frac{172 \sin 8.9°}{\sin 38.7°}$$
$$b \approx 42.34881128$$
$B = 93.7°$, $C = 47.6°$, $b = 274.5$; $B = 8.9°$, $C = 132.4°$, $b = 42.3$

52. Since 57° < 90°, consider Case I.
$b \sin A = 19 \sin 57°$
$b \sin A \approx 15.93474074$
12 < 15.9; no solution

53. Since 29° < 90°, consider Case I.
$c \sin A = 15 \sin 29°$
$c \sin A \approx 7.272144304$
12 > 7.3; 2 solutions
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{12}{\sin 29°} = \frac{15}{\sin C}$$
$$\sin C = \frac{15 \sin 29°}{12}$$
$$C = \sin^{-1}\left(\frac{15 \sin 29°}{12}\right)$$
$$C \approx 37.30170167$$
$180 - \propto \approx 180° - 37.3°$ or 142.7°
Solution 1
$B \approx 180° - (29° + 37.3°)$ or 113.7°
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 29°} \approx \frac{b}{\sin 113.7°}$$
$$b \approx \frac{12 \sin 113.7°}{\sin 29°}$$
$$b \approx 22.6647614$$
Solution 2
$B \approx 180° - (29° + 142.7°)$ or 8.3°
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 29°} \approx \frac{b}{\sin 8.3°}$$
$$b \approx \frac{12 \sin 8.3°}{\sin 29°}$$
$$b \approx 3.573829815$$
$B = 113.7°$, $C = 37.3°$, $b = 22.7$;
$B = 8.3°$, $C = 142.7°$, $b = 3.6$

54. Since 45° < 90°, consider Case I.
83 > 79; 1 solution
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{83}{\sin 45°} = \frac{79}{\sin B}$$
$$\sin B = \frac{79 \sin 45°}{83}$$
$$B = \sin^{-1}\left(\frac{79 \sin 45°}{83}\right)$$
$$B \approx 42.30130394$$
$$C \approx 180° - (45° + 42.3°) \text{ or } 92.7°$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{83}{\sin 45°} \approx \frac{c}{\sin 92.7°}$$
$$c \approx \frac{83 \sin 92.7°}{\sin 45°}$$
$$c \approx 117.2495453$$
$B = 42.3°$, $C = 92.7°$, $c = 117.2$

55. $a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = 40^2 + 45^2 - 2(40)(45) \cos 51°$
$a^2 \approx 1359.446592$
$a \approx 36.87067388$
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{36.9}{\sin 51°} \approx \frac{40}{\sin B}$$
$$\sin B \approx \frac{40 \sin 51°}{36.9}$$
$$B \approx \sin^{-1}\left(\frac{40 \sin 51°}{36.9}\right)$$
$$B \approx 57.39811237$$
$C \approx 180° - (51° + 57.4)$ or 71.6°
$a = 36.9$, $B = 57.4°$, $C = 71.6°$

56. $b^2 = a^2 + c^2 - 2ac \cos B$
$b^2 = 51^2 + 61^2 - 2(51)(61) \cos 19°$
$b^2 \approx 438.9834226$
$b \approx 20.95193124$
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{21.0}{\sin 19°} \approx \frac{51}{\sin A}$$
$$\sin A \approx \frac{51 \sin 19°}{21.0}$$
$$A \approx \sin^{-1}\left(\frac{51 \sin 19°}{21.0}\right)$$
$$A \approx 52.4178316$$
$C \approx 180° - (52.4° + 19°)$ or 108.6°
$b = 21.0$, $A = 52.4°$, $C = 108.6°$

57.
$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$20^2 = 11^2 + 13^2 - 2(11)(13) \cos C$$
$$\frac{20^2 - 11^2 - 13^2}{-2(11)(13)} = \cos C$$
$$\cos^{-1}\left(\frac{20^2 - 11^2 - 13^2}{-2(11)(13)}\right) = C$$
$$112.6198649 \approx C$$
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{11}{\sin A} \approx \frac{20}{\sin 112.6°}$$
$$\sin A \approx \frac{11 \sin 112.6°}{20}$$
$$A \approx \sin^{-1}\left(\frac{11 \sin 112.6°}{20}\right)$$
$$A \approx 30.51023741$$
$B \approx 180° - (30.5° + 112.6°)$ or 36.9°
$A = 30.5$, $B = 36.9°$, $C = 112.6°$

58. $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 42^2 + 6.5^2 - 2(42)(6.5) \cos 24°$

$b^2 \approx 1307.45418$

$b \approx 36.15873588$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{36.2}{\sin 24°} \approx \frac{6.5}{\sin C}$

$\sin C \approx \frac{6.5 \sin 24°}{36.2}$

$C \approx \sin^{-1}\left(\frac{6.5 \sin 24°}{36.2}\right)$

$C \approx 4.192989407$

$A \approx 180° - (24° + 4.2°)$ or $151.8°$

$b = 36.2, A = 151.8°, C = 4.2°$

Page 339 Applications and Problem Solving

59a. $\sin \theta = \frac{8}{12}$

$\theta = \sin^{-1} \frac{8}{12}$

$\theta \approx 41.8°$

59b. $\cos \theta = \frac{x}{12}$

$\cos 41.8° \approx \frac{x}{12}$

$12 \cos 41.8° \approx x$

$8.94427191 \approx x$

about 8.9 ft

60a. $x^2 = 4.5^2 + 8.2^2 - 2(4.5)(8.2) \cos 32°$

$x^2 \approx 24.9040505$

$x \approx 5.0$ mi

60b. $\frac{8.2}{\sin \theta} \approx \frac{5.0}{\sin 32°}$

$\sin \theta \approx \frac{8.2 \sin 32°}{5.0}$

$\theta \approx \sin^{-1}\left(\frac{8.2 \sin 32°}{5.0}\right)$

$\theta \approx 60.54476292$

$180 - \theta \approx 180 - 60.5$ or about $119.5°$

Page 339 Open-Ended Assessment

1. $K = \frac{1}{2} ab \sin C$

$125 = \frac{1}{2}ab \sin 35°$

$435.86 \approx ab$

Sample answer: about 40 cm and 10.9 cm

2a. Sample answer: $a = 10, b = 24, A = 30°$; $10 < 24, 10 < 24 \sin 30°$

2b. Sample answer: $b = 18$; $10 < 18, 10 > 18 \sin 30°$

Chapter 5 SAT & ACT Preparation

Page 341 SAT and ACT Practice

1. There are several ways to solve this problem. Use the Pythagorean Theorem on the large triangle.

$(2y + 3y)^2 = 4^2 + 3^2$

$(5y)^2 = 16 + 9$

$25y^2 = 25$

$\frac{25y^2}{25} = \frac{25}{25}$

$y^2 = 1$

$y = \pm 1$

Since y is a length, use only the positive root. Another method is to use the Triangle Inequality Theorem. The hypotenuse must be shorter than the sum of the lengths of the other two sides.

$5y < 3 + 4$

$5y < 7$

Which of the answer choices make this inequality true?

$5(1) = 5 < 7$

$5(2) = 10 > 7$

The correct choice is *A*.

2. If you recall the general form of the equation of a circle, you can immediately see that this equation represents a circle with its center at the origin.

$(x - h)^2 + (y - k)^2 = r^2$

If you don't recall the equation, you can try to eliminate some of the answer choices. Since the equation contains squared variables, it cannot represent a straight line. Eliminate choice D. Similarly, eliminate choice E. Since both the x and y variables are squared, it cannot represent a parabola. Eliminate choice C. The choices remaining are circle and ellipse. This is a good time to make an educated guess, since you have a 50% chance of guessing correctly. It represents a circle. The correct choice is A.

3. Use factoring and the associative property.

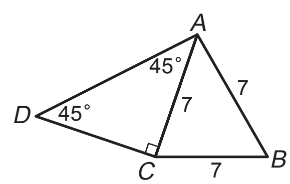
$999 \times 111 = 3 \times 3 \times n^2$

$(9 \times 111) \times 111 = 3 \times 3 \times n^2,$

$3 \times 3 \times (111)^2 = 3 \times 3 \times n^2.$

So n must equal 111. The correct choice is C.

4.



Since $\triangle ABC$ is an equilateral triangle and one side is 7 units long, each side is 7 units long. so $AC = 7$. $\overline{AD}$ is the hypotenuse of right triangle ACD. One leg is 7 units long. One angle is 45°, so the other angle must also be 45°. A 45°−45°−90° triangle is a special right triangle. Its hypotenuse is $\sqrt{2}$ times the length of a leg. (The SAT includes this triangle in the Reference Information at the beginning of the mathematics sections.) The hypotenuse is $7\sqrt{2}$. The correct choice is B.